CONSTITUTIVE MODEL OF SHAPE MEMORY ALLOY FOR CYCLIC DEFORMATION BASED ON ONE-DIMENSIONAL PHASE TRANSFORMATION MODEL

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1. Introduction

Shape memory alloys (SMAs) have unique properties of pseudoelasticity and shape memory effect. Hence they have been applied to commercial products and studied for other new applications [e.g. 1]. However, their mechanical properties are nonlinear including hysteresis and vary depending on temperature, loading frequency, number of cycles and so on. To understand such complicated thermo-mechanical behaviours and to design structural systems including SMA elements optimally, a simple yet reasonably accurate constitutive model is necessary. Although a three-dimensional finite element analysis can be made, its calculation is time-consuming. Moreover, in most of applications an SMA element is in wire, coil or tube and only its one-directional movement is considered. Therefore, lumped parameter models are still useful, especially when calculations must be done for many combinations of parameter values at an early stage of product design.

In our laboratory, several types of constitutive models with an energy-based transformation criterion have been proposed [2-5]. The features of this model are that (1) more than three phases/variants can be considered, (2) rate-dependent effect can be expressed and (3) quantitative analysis can be made.

In this paper this constitutive model is extended to be able to express the cyclic deformation behaviour and the validity of the model is shown by comparing the calculated results with the measured results.

2. Constitutive equations

The constitutive equations, which duplicate the thermodynamic behaviour of shape memory alloys, are comprised of the following three equations [2-5]: the phase transformation criterion,

$$\frac{1}{2}\sigma^2\left(\frac{1}{E_\beta}-\frac{1}{E_\alpha}\right)+\sigma(\varepsilon_\beta-\varepsilon_\alpha)+(s_\beta-s_\alpha)(T-T_{\alpha\leftrightarrow\beta})=\Psi_{\alpha\to\beta}[z_{\alpha 1}]\,, \tag{1}$$

the constitutive equation,

$$\varepsilon=\sigma\sum_\alpha\frac{z_\alpha}{E_\alpha}+\sum_\alpha\varepsilon_\alpha z_\alpha+\alpha_T\left(T-T_s\right), \tag{2}$$

and the energy flow balance equation,

$$C\frac{dT}{dt}+\sum_{\alpha\to\beta}(s_\beta-s_\alpha)T\frac{dz_{\alpha\to\beta}}{dt}+\alpha_T T\frac{d\sigma}{dt}=-h\frac{A}{V}\left(T-T_s\right)+\sum_{\alpha\to\beta}\Psi_{\alpha\to\beta}\frac{dz_{\alpha\to\beta}}{dt}\,. \tag{3}$$

In the equations σ is the stress, E_α is the Young's modulus of phase α, ε_α is the intrinsic strain, s_α is the entropy, T is the temperature of the material, $T_{\alpha\leftrightarrow\beta}$ is the ideal reversible transformation temperature between phase α and phase β, $\Psi_{\alpha\to\beta}$ is the energy required when phase α transforms into phase β due to the dissipation such as internal friction and $z_{\alpha 1}$ is the variable related to the volume fraction of phase α. In the constitutive equation ε is the strain, z_α is the volume fraction of phase α, α_T is the linear coefficient of expansion and T_S is the surrounding temperature. In the energy flow balance equation C is the specific heat capacity, t is the time, $z_{\alpha\to\beta}$ is the volume fraction transforming from phase α to phase β, h is the coefficient of conduction and A/V is the area/volume. In this study we consider not only austenite phase (A) and martensite phase (M) but also austenite phase with residual martensite phase (AR_i), which is often observed during the first several reverse transformations. Accordingly, phase transformation proceeds as A→M→ AR_1→M→AR_2→M, where AR_1 and AR_2 are assumed to have different amount of residual martensite phase.

3. Result

Figure 1 shows a stress-strain curve for the first two and a half of full cycle. Symbols represent measured data and lines are prediction. It is seen that the prediction can capture the residual strain and the reduction of transformation stress well. Figure 2 shows a stress-strain curve for cyclic deformation with increasing strain amplitude. The prediction can capture staircase-like increase of the transformation stress during the second and the third loading phases.

4. Conclusion

One-dimensional phase transformation model was extended so as to express the cyclic deformation behaviour. Comparison in stress-strain curve of the prediction with the measured data showed the validity of the model.

5. References

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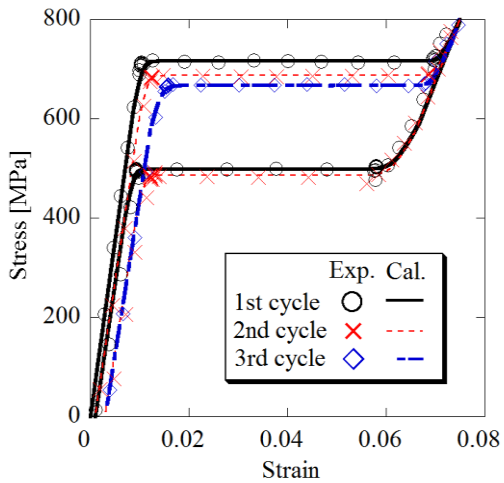

Figure 1. Stress-strain curve for full strain cycles.

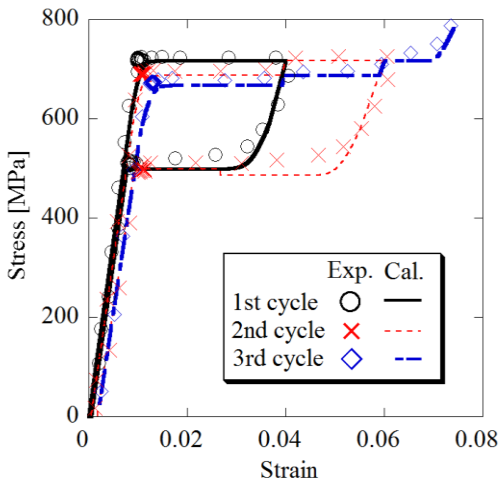

Figure 2. Stress-strain curve for cyclic deformation with increasing strain amplitude.