ON DISPERSION PHENOMENA IN THE FRAMEWORK OF THE FRACTIONAL CONTINNUM MECHNICS

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ABSTRACT

In this paper the dispersion phenomena of a 1D solid is analysed using the Fractional Continuum Mechanics (FCM) approach. The results are compared with the reference dispersion curve of Born-Von Karman (BK) lattice. The fundamental result is that for the length scale in the FCM model comparable with the lattice spacing in BK model the dispersive curve for both formulations is equivalent, whereas the order of fractional continua plays the role of scaling parameter.

1. Introduction

It is well know that structured solids present dispersive behaviour which cannot be captured by the classical local continuum mechanics approaches. A canonical problem in which this can be seen is the wave propagation in the Born-Von Karman (BK) lattice [1]. The problem of covering BK solution based on well-known non-local formulations has been reported in [2].

In this paper the dispersive effects in a 1D structured solid is analysed using the FCM approach previously proposed by Sumelka [3, 4]. The results obtained within this approach have been compared with the reference dispersion curve of BK lattice. It is observed the fractional model can capture the size effects in the dynamic behaviour in a similar way as BK discrete system.

2. Problem formulation

Born-Von Karman lattice

As a reference discrete model, we will consider the Born-Von Karman lattice, a one-dimensional infinite chain of monoatomic particles at sites with linear interactions between nearest neighbours. The equation of motion for particle *j* is

$$M\ddot{u}_j = C\left(u_{j-1} - 2u_j + u_{j+1}\right)$$

which leads to the dispersion relation

$$\bar{\omega} = 2\left|\sin\left(\frac{\bar{k}}{2}\right)\right|$$

where

$$\bar{k} = ka, \quad \bar{\omega} = \frac{\omega a}{c_0}$$

and M is the mass of each particle, u is the displacement, C is the stiffness of the interaction, k is the wavenumber, ω is an angular frequency, a is the lattice spacing and c_0 denotes

$c_0^2 = Ca^2/M$

Fractional Continuum Model

For the case of FCM the dynamic equilibrium for 1D case is [4]

$$E\ell_f^{\alpha-1}\frac{1}{2}\frac{\Gamma(2-\alpha)}{\Gamma(2)}\frac{\partial}{\partial X}\left({}_{X-\ell_f}^{C}D_X^{\alpha}u - {}_X^C D_{X+\ell_f}^{\alpha}u\right) = \rho_0\frac{\partial^2 u}{\partial t^2}$$

where E is the Young's modulus, l_f is the length scale, X is a spatial variable, D denotes the fractional differentiation, α is the order of fractional continua, ρ_0 is reference density and t denotes time.

For the FCM model the dispersion curve has been obtained based on the numerical solution of vibration of 1D fractional body, and applying the Fast Fourier Transform on the obtained signal to capture the fundamental frequency – point in a k vs. ω curve (cf. Fig. 1).

3. Numerical examples and discussion

In Fig. 1 the dispersion curves for FCM, CCM and Born-Von Karman models in the right half of the first Brillouin zone are presented. The curves were obtained under the assumption that length scale in FCM model is equal to the lattice spacing in a discrete BK chain. It should be noticed that the order of fractional continua α plays the role of scaling parameter.

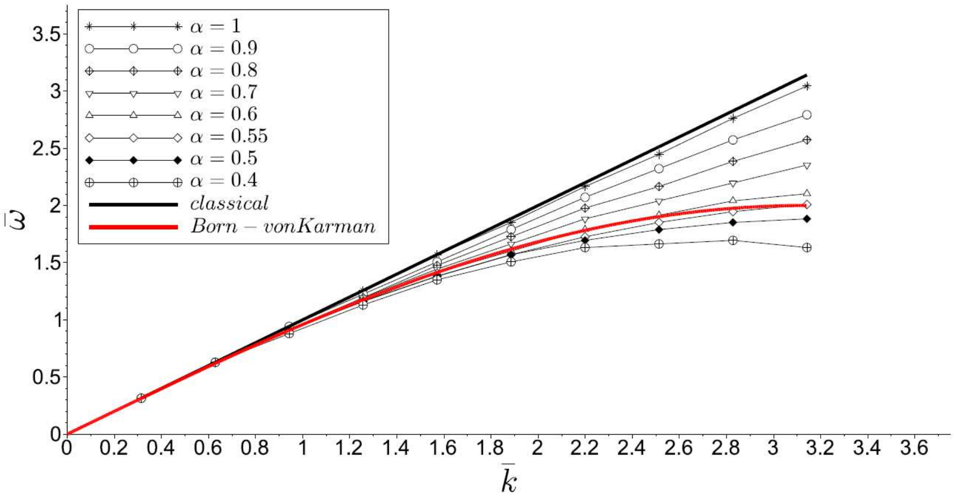

Fig. 1 Dispersion curves for FCM, CCM and Born-Von Karman models in the in the right half of the first Brillouin zone.

4. References

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