MODELLING OF LOCALIZED DAMAGE USING AN ENHANCED EMBEDDED DISCONTINUITY APPROACH: AN OVERVIEW

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1. General

This presentation gives an overview of the research conducted by the author and his co-workers in relation to description of the onset and propagation of localized damage in cohesive-frictional materials. The first part provides a brief outline of the formulation of the problem, illustrated by a series of examples. The propagation of damage is described in terms of an embedded discontinuity approach, which employs volume averaging [1-3]. The approach incorporates the notion of a 'characteristic dimension', which is explicitly defined as the ratio of the area of macrocrack to the selected reference volume, the latter identified with that of a finite element containing the discontinuity. Some numerical examples are provided, which include a four-point bending test as well a mixed-mode fracture test. The results are compared with those obtained using XFEM methodology and/or the available experimental data. For fracture in compression regime, a number of biaxial tests on a sedimentary rock are simulated that highlight the effects of kinematic constraints and the inherent anisotropy of the material on the mechanical characteristics. The next part of this work deals with description of chemo-mechanical interaction in geomaterials and its coupling with the onset and propagation of localized damage. First, a general formulation of the problem is reviewed, which incorporates the framework of chemo-plasticity. Applications of this approach are then presented, which involve description of the intergranular pressure solution in chalk as well as the alkali-aggregate reaction in concrete. The last part of the presentation is focused on the assessment of the size effect in concrete structures subjected to a broad range of loading conditions that include a chemo-mechanical coupling. Numerical examples deal with a series of three-point bending tests as well as compression tests. For continuing alkali-silica reaction, it is demonstrated that by increasing the size of the structure, a spontaneous failure may occur under a sustained load. It is clearly shown that the size effect is associated with propagation of localized damage whose description requires an explicit definition of a characteristic length.

2. Modeling of fracture propagation via a constitutive relation with embedded discontinuity

The onset of localized deformation and the orientation of the localization plane can be defined based on the bifurcation criterion. The damage propagation process is modelled through an enhanced discrete representation of the constitutive law with embedded discontinuity [3]. For a discontinuous motion, the symmetric part of the velocity gradient is defined as

$$\nabla^S \mathbf{v}(\mathbf{x},t) = \nabla^S \hat{\mathbf{v}}(\mathbf{x},t) + \mathcal{H}(\phi)\nabla^S \tilde{\mathbf{v}}(\mathbf{x},t) + \delta(\phi)(\mathbf{n}\otimes\tilde{\mathbf{v}})^S$$

Here, $\hat{\mathbf{v}}$ and $\tilde{\mathbf{v}}$ are two continuous functions, $\mathcal{H}(\phi)$ is the Heaviside step function and $\phi = \phi(\mathbf{x},t)$ is the level-set function that represents the geometry of the crack. Note that $\nabla^S \mathcal{H}(\phi) = \delta(\phi)\nabla\phi$, where δ is the Dirac delta function and the gradient of a level-set function represents the normal to the surface, i.e. $\nabla\phi = \mathbf{n}$. Taking the volume average of the last term in the equation above, which is associated with localized deformation along the macrocrack, leads to a resolution of the total strain rate into two elementary parts. The first one, denoted as $\dot{\hat{\boldsymbol{\varepsilon}}}$, is associated with the intact part of the reference volume, while the other one, referred to as $\dot{\tilde{\boldsymbol{\varepsilon}}}$, represents the discontinuous motion along the crack averaged over this volume, i.e.

$$\dot{\boldsymbol{\varepsilon}} = \dot{\hat{\boldsymbol{\varepsilon}}} + \dot{\tilde{\boldsymbol{\varepsilon}}} \quad \textit{where} \quad \dot{\tilde{\boldsymbol{\varepsilon}}} = \chi(\mathbf{n}\otimes\dot{\mathbf{g}})^S$$

In the expression above, $\dot{\mathbf{g}}$ is the velocity discontinuity along the interface, i.e. $\dot{\mathbf{g}} = [\![\mathbf{v}]\!] = [\![\mathcal{H}]\!]\,\tilde{\mathbf{v}}$ and

$\chi = \Delta a / \Delta v$ where Δa represents the surface area of the crack within the considered volume Δv. By imposing now the continuity of traction along the interface, the constitutive law is obtained, viz.

$$\dot{\boldsymbol{\sigma}} = \tilde{\mathbb{D}} : \dot{\boldsymbol{\varepsilon}} ; \quad \tilde{\mathbb{D}} = \mathbb{D} - \mathbb{D} : \mathbb{E} : \mathbb{D}, \quad \mathbb{E} = \chi \, \mathbf{n} \otimes (\mathbf{K} + \chi \, \mathbf{n} \cdot \mathbb{D} \cdot \mathbf{n})^{-1} \otimes \mathbf{n}$$

where $\mathbf{K}$ is the tangential operator which defines the interfacial properties. The latter can be identified by invoking, for example, a plasticity framework that incorporates strain-softening and relates the velocity discontinuity to the rate of traction. In the numerical implementation, the volume of the finite element is perceived to as a reference volume and the macrocrack is traced in a discrete manner by using the level-set method. Within this scheme, the failure/bifurcation criterion is checked in the candidate elements adjacent to the crack tip and, if met, the average direction of propagation is established. In 3D case, in order to avoid numerical difficulties associated with an abrupt change in the crack surface orientation, a crack smoothening algorithm has been implemented.

In order to illustrate the framework a series of numerical examples are provided. The first set of examples deals with simulation of experimental tests that involve four-point bending of a notched concrete beam under the action of two independent actuators as well as a mixed mode cracking. The results based on embedded discontinuity approach are directly compared with XFEM simulations. The predictions from both these methodologies are virtually identical and are consistent with the experimental data; this gives the advantage to the former scheme in view of its simplicity in the numerical implementation. The follow up example involves numerical analysis of a shear band formation in biaxial tests conducted on Tournemire argillite. This material has a strong inherent anisotropy that is described using a microstructure tensor approach outlined in ref.[4]. The effects of boundary conditions, orientation of bedding planes, and mesh-sensitivity of the approach are studied. It is demonstrated that friction between loading platens can play an important role in the process of evolution of damage and may significantly affect the strength characteristics that are commonly perceived as a material property.

The next part involves modeling of the mechanical response associated with a chemo-mechanical interaction and the evolution of localized damage resulting from this coupling. First, a chemo-plasticity framework is outlined for which the kinetics of the reaction is examined in the context of the intergranular pressure solution in certain types of rocks (e.g., chalk) as well as an alkali –silica reaction in concrete/reinforced concrete. For the former case, the micromechanics that governs the underlying evolution law and the associated time-dependent behaviour is examined. The last topic covered deals with the assessment of deterministic size effect in concrete structures experiencing different fracture modes that include tensile failure, formation of macrocracks in compression regime as well as damage due to chemical interaction. It is clearly shown that the size effect is triggered by the onset and propagation of localized damage; the latter associated with strain-softening, whose rate is controlled by a 'characteristic length' that needs to be uniquely defined. For continuing alkali-silica reaction, it is demonstrated that, by increasing the size of the structure, a spontaneous failure may occur under a sustained load.

References

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