SOLUTION OF THE THREE FORCE PROBLEM IN A CASE OF TWO FORCES BEING MUTUALLY ORTHOGONAL

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1. Introduction

The three force problem (3FP), till now unsolved in general, is one of the most important task of Michell truss theory. The problem is formulated as follows: *find the lightest fully stressed truss transmitting three self-equilibrated co-planar forces, assuming the feasible domain is the whole plane*. The significant class of solutions of 3FP was derived by Chan [1] and utilized by Sokół and Lewiński [2,3,4] and Sokół and Rozvany [5] to obtain the optimal truss with two point loads between supports. Some geometrical aspects of the optimal solution of 3FP for discretized Michell trusses was investigated by Mazurek [6]. Recently, the present authors obtained exact solutions of simply supported Michell trusses generated by lateral point load and restricted to a unit square domain [7]. In the present paper we extend the class of solutions by enhancing the feasible design domain and the position of the point load to the whole plane. However, as before, we assume that two of the three forces are mutually orthogonal, so the problem can be stated as in Fig. 1. Note, that formally, the supports can be replaced by the appropriate reaction forces but it is convenient to include them to eliminate rigid body motions and to obtain a uniquely defined displacement field.

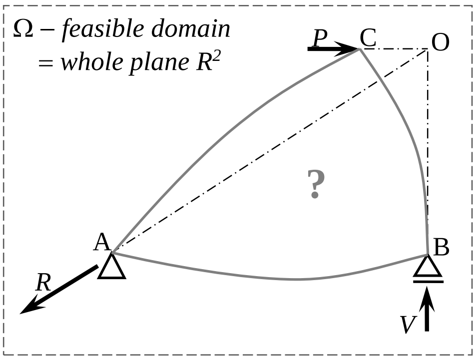

Figure 1. The three force problem in a case of two forces being mutually orthogonal.

2. Numerical prediction of the optimal layout

An exact analytical solution of any Michell problem is very hard to obtain due to the necessity of solving a hyperbolic differential system and requires a good prediction of the proper layout, which usually has to be divided into appropriate regions. This difficult task can be performed in reliable and effective way using the adaptive ground structure method. This is not the main topic of the paper and will not be given in detail here (a description of the latest, most powerful version of this method and related references can be found in [8]). Nevertheless, it should be pointed out that this method allows the use of very dense ground structures which reduce the discretization error, and thus makes it possible to obtain very accurate approximations of the optimal solutions to be constructed.

The problem investigated in the present paper (see Fig. 1) can be regarded as a two-parameter family of solutions depending of the position of the applied lateral force (x_C and y_C coordinates of point C). After performing numerical tests for different positions of load, we can recognize and classify the possible optimal layouts which then can be confirmed by exact analytical formulae.

3. Adjusting the optimal solution by exact analytical formulae

A strict mathematical procedure to derive exact analytical formulae describing geometry and virtual displacement field of Michell continuum for a presumed optimal topology has been described in detail in several papers (cf. reference lists given in [1-7]). How to apply this procedure to solve the problem posed in Fig. 1 will be discussed in the full length paper, and due to a limited space of the abstract, we present only one selected numerical result and its analytical counterpart, see Fig. 2.

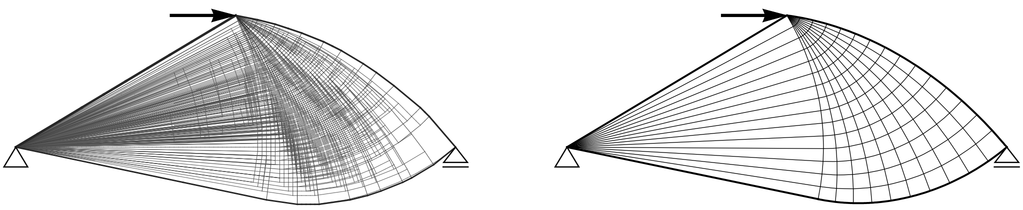

Figure 2. Exemplary solution of 3FP: a) numerical and b) analytical result.

4. Final remarks

Among Michell problems still unsolved, the three force problem is probably the most important. If extended to spatial case of four force problem, it may deliver patterns for new designs, especially those that can be produced by additive manufacturing, with using metallic components, see Smith et al. [9].

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