A STUDY OF RATE-DEPENDENT AND RATE-INDEPENDENT REGULARIZATION OF CRYSTAL PLASTICITY AT FINITE STRAINS

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1. Introduction

In this work, we aim to compare two regularization schemes of crystal plasticity: rate-dependent [1, 2] and rate-independent [3] regularization. Obtained numerical results are confronted with experimental data [4].

2. Regularization approaches

Single crystal plasticity at finite strains is based on a well-established kinematics framework. It employs the elastic-plastic multiplicative split of the deformation gradient $\mathbf{F}$, and the plastic velocity gradient $\mathbf{L}^p$ is composed of the contributions of individual slip systems, thus

$$\mathbf{F} = \mathbf{F}^e\mathbf{F}^p, \qquad \mathbf{L}^p = \dot{\mathbf{F}}^p\mathbf{F}^{-p} = \sum_{\alpha=1}^{N_s} \dot{\gamma}_\alpha \mathbf{s}_\alpha \otimes \mathbf{m}_\alpha, \tag{1}$$

where $\dot{\gamma}_\alpha$ is slip rate on slip system α, and $\mathbf{m}_\alpha$ and $\mathbf{s}_\alpha$ denote slip system's normal and slip direction, respectively.

In the first regularization approach, there is no yield surface, and all slip systems are assumed active. The slip rates $\dot{\gamma}_\alpha$ are specified by a nonlinear function (power-law) of the corresponding ratio of the resolved shear stress τ_α to the critical resolved shear stress τ_α^c,

$$\dot{\gamma}_\alpha = \dot{\gamma}_0 \,\mathrm{sign}\tau_\alpha \left(\frac{|\tau_\alpha|}{\tau_\alpha^c}\right)^m, \qquad \tau_\alpha = \mathbf{M} \cdot (\mathbf{s}_\alpha \otimes \mathbf{m}_\alpha), \tag{2}$$

where $\dot{\gamma}_0$ is a constant material parameter called reference slip rate, and it is identical for all slip systems. $\mathbf{M}$ is the Mandel stress tensor that is work-conjugate with $\mathbf{L}^p$. This regularization yields a rate-dependent elastic-viscoplastic model.

In a rate-independent approach, a single yield surface $F \leq 0$ is introduced, and plastic velocity gradient is defined by an associated flow rule,

$$F(\mathbf{M}, \tau_\alpha^c) = \left(\sum_{\alpha=1}^{N_s} \left(\frac{\tau_\alpha}{\tau_\alpha^c}\right)^{2n}\right)^{1/2n} - 1 \leq 0, \quad \mathbf{L}^p = \dot{\lambda}\frac{\partial F}{\partial \mathbf{M}}, \quad \dot{\lambda} \geq 0, \quad \dot{\lambda}F = 0. \tag{3}$$

It can be checked that $\mathbf{L}^p$ defined above is of the form $(1)_2$. Plastic multiplier $\dot{\lambda}$ is determined from the consistency condition $\dot{F} = 0$.

3. Results

As shown by Rice [5], loading of an FCC crystal along certain crystallographic directions results in a simplified plane-strain model with three effective slip systems instead of twelve in the full 3D model. This simplified 2D model has been used in the present study.

Numerical simulations of wedge indentation into a Ni single crystal have been performed in order to investigate differences between viscoplastic (rate-dependent) and rate-independent regularization. Geometrical data and material parameters have been taken from [4]. Figure 1 shows a contour plot of the lattice rotation computed using the rate-independent model. It is compared to experimental measurements taken from [4]. Agreement between experimental result and numerical calculations is acceptable and the major features of lattice rotation are correctly reproduced.

In Fig. 2, detailed graphs presenting lattice rotation along $x_2 = const$ line in the deformed configuration. In the right graph in Fig. 2, comparison of the two regularization schemes is presented. No significant difference in lattice rotation is observed.

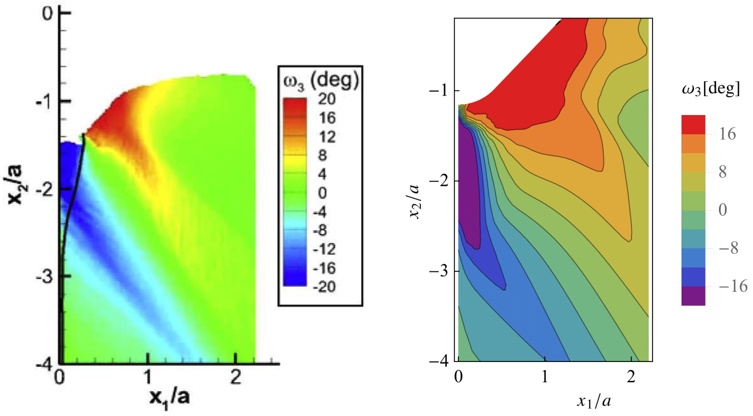

Figure 1. Wedge indentation in Ni single crystal: experimental results [4] (left) and numerical results calculated using the rate-independent model, $n = 20$ (right).

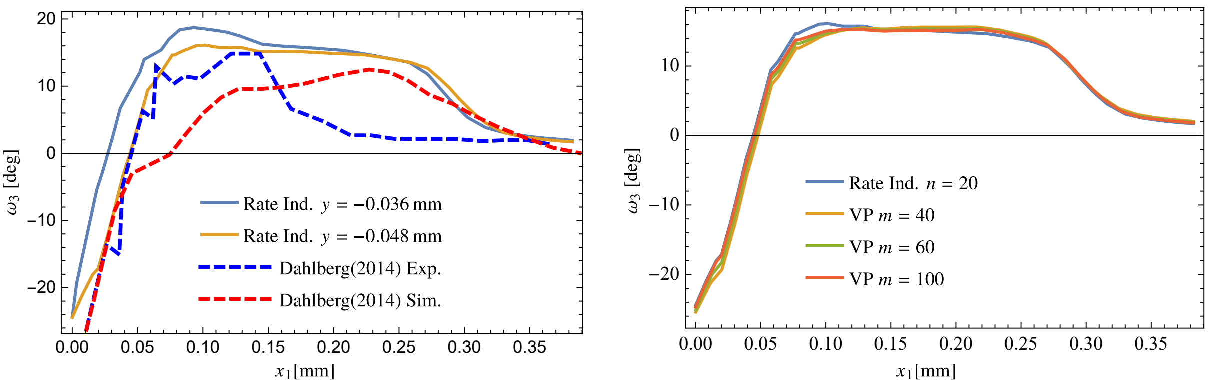

Figure 2. Comparison of the obtained lattice rotations with experimental and numerical results from [4].

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