

# SIMULATIONS OF THERMAL SOFTENING IN LARGE STRAIN THERMOPLASTICITY WITH DEGRADATION

**B. Wcisło<sup>1</sup>, J. Pamin<sup>1</sup> and A. Menzel<sup>2</sup>**

<sup>1</sup>*Cracow University of Technology, Poland*

<sup>2</sup>*Technische Universität Dortmund, Germany / Lund University, Sweden*

## 1. Introduction

When a material experiences extreme loading it initially deforms uniformly and from some point strains localize in a narrow zone while the rest of the material unloads. The phenomenon called strain localization, which is a precursor of material fracture, can have three sources: material degradation, geometrical softening or temperature-induced softening. In this paper special attention is focused on the numerical analysis of thermal softening; however the geometrical and material instabilities are approached simultaneously. The thermal softening is understood here as a reduction of the yield strength due to the increase of temperature. It can take various forms, see e.g. [1] or [2], and it manifests itself as a reduction of the total yield strength, hardening function or individual material parameters.

## 2. Model description

In the presented large strain model a full thermomechanical coupling is adopted, i.e. the model includes thermal expansion, dependence of material parameters on temperature (degradation of Young modulus and yield strength), plastic self-heating and the influence of deformation on the heat flux. Non-stationary Fourier heat transport through the isotropic material is considered.

The analysed thermomechanical model is based on the multiplicative decomposition of the deformation gradient  $\mathbf{F} = \mathbf{F}^\theta \mathbf{F}^e \mathbf{F}^p$ , where  $\mathbf{F}^\theta = \exp[\alpha_T(T - T_0)]\mathbf{I}$  is a part related to thermal expansion ( $\alpha_T$  denotes the coefficient of linear thermal expansion,  $\mathbf{I}$  is the second order identity tensor,  $T_0$  and  $T$  are the reference and absolute temperatures respectively),  $\mathbf{F}^e$  is an elastic contribution and  $\mathbf{F}^p$  involves the irreversible (plastic) deformation. The state of the material is described by the Helmholtz free energy potential that is assumed in the following decoupled form [1]

$$(1) \quad \psi(\mathbf{b}^e, T, \gamma) = \psi^e(\mathbf{b}^e) + \psi^\theta(T) + \psi^p(\gamma)$$

which distinguishes the elastic, thermal and plastic parts. The quantity  $\mathbf{b}^e = \mathbf{F}^e(\mathbf{F}^e)^T$  denotes the elastic left Cauchy-Green tensor and  $\gamma$  is a scalar plastic strain measure.

The yield function that governs the plastic regime is assumed in the following form

$$(2) \quad F_p(\boldsymbol{\tau}, \gamma, T) = f(\boldsymbol{\tau}) - \sqrt{2/3}\sigma_y(\gamma, T) \leq 0$$

The equivalent stress function  $f(\boldsymbol{\tau})$  is a Kirchhoff stress measure (e.g. Huber-Mises-Hencky) and  $\sigma_y(\gamma, T)$  denotes the yield strength which includes strain hardening, thermal softening and, optionally, a measure of degradation of plastic properties as in [3]. If the last phenomenon is incorporated in the model, the description is enhanced with the gradient regularization presented in [3].

The yield strength functions with thermal softening incorporated are for simplicity limited to linear temperature-dependence and can for instance have one of the following two forms:

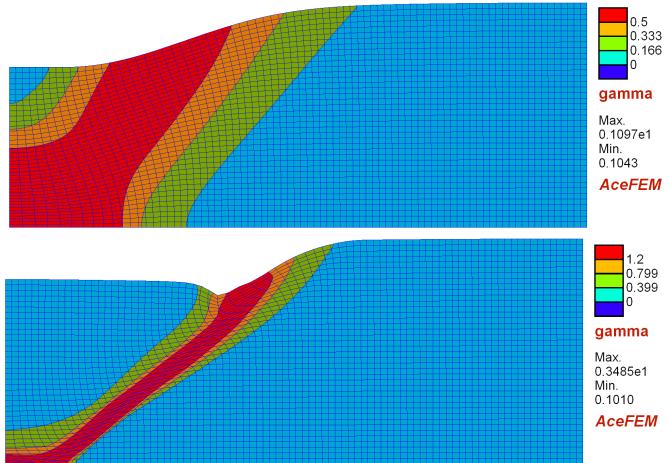
$$(3) \quad \sigma_y(\gamma, T) = [Y_0 + H\gamma + (Y_\infty - Y_0)(1 - e^{-\delta\gamma})][1 - H_T(T - T_0)]$$

$$(4) \quad \sigma_y(\gamma, T) = Y_0 + H(T)\gamma + (Y_\infty(T) - Y_0)(1 - e^{-\delta\gamma})$$

In the above equations  $Y_\infty(T) = Y_\infty[1 - H_{T1}(T - T_0)]$ ,  $H(T) = H[1 - H_{T2}(T - T_0)]$ , moreover  $Y_0$ ,  $H$ ,  $Y_\infty$ ,  $\delta$ ,  $H_T$ ,  $H_{T1}$  and  $H_{T2}$  are material parameters.

### 3. Implementation and numerical tests

The numerical simulations are performed using symbolic-numerical packages *Ace* that work in *Wolfram Mathematica* environment. The user-supplied subroutines are developed for the obtained two- or three-field finite element formulation. A series of computational tests for an elongated plate in plane strain conditions are performed. Special attention is focused on the influence of the adopted form of thermal softening on simulation results. Moreover, different boundary conditions for thermal field are taken into account (convection, insulation) and different finite elements are tested. This last aspect is illustrated in Figure 1 where deformed meshes with the plastic strain distribution are depicted for standard elements with linear interpolation of all fields and for locking-free elements with *F-bar* modification, see [4]. The results reveal that the material response can differ strongly not only depending on the specific form of material functions (e.g. defining thermal softening), but also depending on the finite element quality.



**Figure 1.** Deformed meshes with temperature distribution for elements without *F-bar* (top) and with *F-bar* (bottom) – nonadiabatic local thermoplasticity with material and thermal softening as in Eq. (3)

### 4. References

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