A NEW ASYMPTOTIC-TOLERANCE MODEL OF DYNAMIC PROBLEMS FOR THIN TRANSVERSALLY GRADED CYLINDRICAL SHELLS

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1. Formulation of the problem

The objects of consideration are thin linearly elastic Kirchhoff-Love–type open circular cylindrical shells having a tolerance-periodic microstructure in circumferential direction. It means that *on the microscopic level*, the shells under consideration consist of many small elements. These elements, called *cells* and treated as thin shells, are not repeated, in contrast to cylindrical shells with a periodic structure. It is assumed that the adjacent cells are nearly identical, but the distant elements can be very different. The length dimension of the cell in circumferential direction is described by constant parameter λ (a shell with *a uniform cells distribution*). It is assumed that *microstructure length parameter* λ is very large compared with the maximum shell thickness and very small as compared to the midsurface curvature radius as well as the length dimension of the shell midsurface in circumferential direction. An example of such shell is shown in Fig. 1.

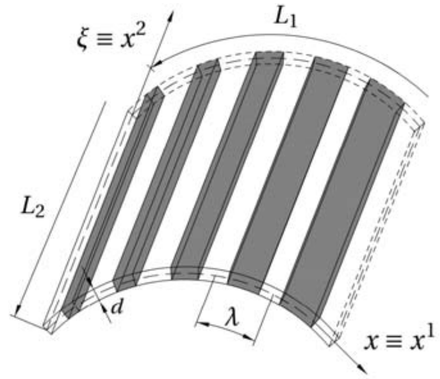

Figure 1. An example of a shell with a tolerance-periodic microstructure.

On the microscopic level, the geometrical, elastic and inertial properties of these shells are determined by highly oscillating non-continuous *tolerance-periodic functions* in *x*, where *x* stands for coordinate parametrizing the shell midsurface in circumferential direction. By *tolerance periodic functions* we shall mean functions which in every cell can be approximated by periodic functions in *x*.

On the other hand, *on the macroscopic level*, under assumption that the adjacent cells are almost identical, the averaged (effective) properties of the shells are described by functions being *smooth* and *slowly varying* along circumferential direction. It means that the tolerance-periodic shells under consideration can be treated as made of *functionally graded materials* (FGM), cf. [2], and called *functionally graded shells*. Moreover, since effective properties of the shells are graded in direction normal to interfaces between constituents, this gradation is referred to as *the transversal gradation*. At the same time, the shells have constant properties in axial direction.

The dynamic problems of such shells are described by partial differential equations with highly oscillating, tolerance-periodic, non-continuous coefficients, so these equations cannot be a proper tool to investigate special engineering problems of such shells. To obtain averaged equations with continuous and slowly varying coefficients, a lot of different approximate modelling methods have been proposed. FGM-type structures are often analysed in the framework of averaging approaches for macroscopically homogeneous structures, e.g. periodic. Some of these methods are presented in [2]. Between them we can distinguish models based on the asymptotic homogenization, cf. [1]. Unfortunately, in the models of this kind *the effect of a microstructure size* (called *the length-scale effect*) on the overall shell behaviour is neglected. This effect can be taken into account using *the tolerance averaging technique*, cf. [4]. Some applications of this method to the modelling of various periodic structures are shown in many works. The extended list of papers and books on this topic can be found in [3, 4]. Using the tolerance averaging procedure, the length-scale effect in dynamic and stability problems for micro-periodic cylindrical shells was analysed by Tomczyk in monograph [3]. In the last years the tolerance modelling was adopted for mechanical and thermomechanical problems of functionally graded structures, cf. [4].

The aim of this contribution is to formulate *a new averaged combined asymptotic-tolerance model for the analysis of selected dynamic problems for the tolerance-periodic cylindrical shells under consideration.* Contrary to starting equations with highly oscillating, non-continuous, tolerance-periodic coefficients, *governing equations of the proposed model have continuous and slowly varying coefficients depending also on a microstructure size* λ. Hence, this model makes it possible to describe the effect of a length scale on the dynamic shell behaviour. The model will be derived applying *the combined asymptotic-tolerance modelling technique* given by Woźniak in [4].

2. Model equations and concluding remarks

Governing equations of the combined model, proposed here, consist of *the macroscopic model equations* (with continuous and slowly varying coefficients being independent of a cell size λ) formulated by means of *the consistent asymptotic procedure*, [4], which are combined with *the superimposed microscopic model equations* (with continuous and slowly varying coefficients depending also on a microstructure size λ) derived by applying *the tolerance modelling technique*, [4], and under assumption that in the framework of the macroscopic model the solutions to the problem under consideration are known.

The resulting combined model equations are uniquely determined by the highly oscillating tolerance-periodic *fluctuation shape functions* representing oscillations inside a cell, which have to be known in every problem under consideration. In dynamic problems, these functions represent either the principal modes of free cell vibrations or physically reasonable approximation of these modes. An important advantage of the combined model is that under special condition imposed on the fluctuation shape functions, *it makes it possible to separate the macroscopic description of some special problems from their microscopic description*. It means that in the framework of the combined model we can study the shell micro-dynamics (e.g. free micro-vibrations depending on λ, wave propagation related to micro-fluctuations of the shell displacements) independently of the shell macro-dynamics. The combined model equations also describe *certain time-boundary and space-boundary phenomena* strictly related to the specific form of initial and boundary conditions imposed on micro-fluctuations of the shell displacements being unknowns of the superimposed microscopic model.

3. References

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