BRINKMAN'S FLOW THROUGH POROUS ELASTIC MEDIA: AN ASYMPTOTIC APPROACH

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1. General

The aim of present contribution is to study flow of Stokesian fluid in a linear elastic porous medium, and to derive macroscopic equations of Brinkman's type.

The analysis of the Stokesian flow through porous medium is commonly realized under the assumption of the incompressibility of the fluid and the ideal rigidity of a porous structure (a skeleton). However, in such analysis an experimental evidence is neglected that the compressibility of fluid (water) is always smaller than that of the skeleton material. The compressibility of the water at $25^{\circ}\mathrm{C}$ equals $4.6 \times 10^{-10}\mathrm{Pa}^{-1}$, while the compressibility of the dense sand or sandy gravel is of the order of $1 \times 10^{-8}\mathrm{Pa}^{-1}$, while this of the hard rocks $\approx 3 \times 10^{-10}\mathrm{Pa}^{-1}$. This means that in many important for the practice cases the elasticity of skeleton should be accounted for.

The isotropic seepage of a viscous Newtonian fluid through a porous skeleton is described at a macroscopical scale (a scale large with respect to the pore dimensions) by Darcy's law. The vector field **v** denotes the velocity of the fluid defined as the mean rate of flow through a surface element of unit area. This law patterned on others transport equation (Fourier's, Ohm's, Fick's) does not render aptly the specificity of the flow at peripheries of porous medium. A basic difficulty is that any viscous shear tensor can be introduced in relation to it, as the viscous shearing in Darcy's flow is neglected. Related to this objection are difficulties in posing the boundary conditions, for example for problems in which the fluid flows through porous medium and adjoining empty space, cf. [1] - [3].

2. Brinkman's equation

For these reasons H. C. Brinkman proposed to supplement Darcy's law in the additional term containing the Laplacian of the fluid velocity $\mathbf{v}$. He considered the incompressible fluid

$$(1) \qquad \nabla \cdot \mathbf{v} = 0$$

and suggested the following equation

$$(2) \qquad \nabla p = -\frac{\eta}{K}\mathbf{v} + \eta' \Delta \mathbf{v}$$

where p is the pressure field in the fluid, while K is the permeability of porous medium. The coefficient η is the fluid viscosity and the coefficient η' is a modified fluid viscosity which may be different from η. This equation for low values of K is approximated by Darcy's equation

$$(3) \qquad \mathbf{v} = -\frac{K}{\eta}\nabla p$$

while for high values of K Stokes' equation (it is Navier-Stokes equation with the inertial terms neglected) is obtained.

$$(4) \qquad \nabla p = \eta \Delta \mathbf{v}$$

The boundary conditions at the interface between porous medium and free space filled by the fluid may be derived for Brinkman's equation (2). One easily observe that the first term (resulting from

Darcy's law) at RHS is negligible in comparison to the normal σ_{nn} and shearing σ_{nt} stresses. Therefore the following components of stress should be continuous

$$\sigma_{nn} = -p + 2\eta' \frac{\partial \mathbf{v}_n}{\partial \mathbf{n}} \quad \text{and} \quad \sigma_{nt} = \eta' \left(\frac{\partial \mathbf{v}_n}{\partial \mathbf{t}} + \frac{\partial \mathbf{v}_t}{\partial \mathbf{n}} \right) \tag{5}$$

where $\mathbf{n}$ indicates the normal direction and $\mathbf{t}$ the tangential direction. Assuming (2) to be valid in transition region the tangential velocity component is continuous at the interface.

3. Equations of micro-periodic porous medium

Let Ω be an open bounded and connected domain with the boundary $\partial\Omega$. The domain Ω has an εY - periodic structure. For a fixed $\varepsilon > 0$ all the relevant quantities are marked by the superscript ε. Let $\mathbf{u}^\varepsilon$ and $\mathbf{v}^\varepsilon$ be the fields of displacement in the elastic skeleton and the velocity in Ω_F^ε, respectively. By p^ε we denote the pressure in the fluid. The interface fluid-solid is denoted by Γ^ε. On the interface of fluid-solid the continuity of normal stresses and of velocities is imposed.

4. Separation of scales, asymptotic expansions and homogenisation

If l and L are the characteristic lengths of the local and the macroscopic scales their ration should obey the inequality $\varepsilon = l/L \ll 1$. This condition is required to obtain Darcy's law, cf. [4] - [7]. If the separation of scales is not so distinct, it is, if we have only

$$\varepsilon = \frac{l}{L} < 1 \tag{6}$$

then the modification of Darcy's equation presented by Eq.(2) can be obtained, cf. [5].

The asymptotic expansions of mechanical fields in both, the fluid and the skeleton permits to obtain Brinkman's term of relative weight $\mathcal{O}(\varepsilon^3)$, as in [5], but our equation involves the motion of the elastic skeleton, cf. [6]. The problem discussed in [7] is solved as an example.

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5. References

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