1. Introduction

Tensegrities are defined as cable-strut structures consisting of isolated compressed elements inside a continuous net of tensioned members [5,6]. Node configuration of these structures ensures occurrence of infinitesimal mechanisms that are balanced with self-stress states. Tensegrity structures are complicated regarding both their geometry and mechanical properties. In order to understand their actual properties and identify features of the structure as a whole, a continuum model is considered.

The present paper focuses on application of a continuum theory in regard to a three-dimensional tensegrity plate-like structure. As a result, a two-dimensional plate theory is built to describe mechanics of a space tensegrity structure. The first step of the proposed modelling is selection of an orthotropic repetitive segment, which is taken out from the tensegrity plate-like structure. Then, the selected representative segment undergoes numerical homogenization [3,1]. By comparing the elastic strain energy from FEM truss formulation with the energy of a solid, a continuum model of the segment is obtained. The homogeneous segments are afterwards joined together to create a three-dimensional orthotropic continuum, which includes the effect of self-stress [4]. After applying the assumptions of plate theory for moderately thick plates and integration over the thickness, a two-dimensional plate model is obtained for both membrane and bending deformations. The model includes the effect of self-stress that was initially applied to the tensegrity plate-like structure.

2. Six-parameter plate theory

Mathematical model of the tensegrity plate-like is a six-parameter flat shell theory [2] with the curvature tensor \( b_{\alpha \beta} = 0 \). Let us consider a flat shell of thickness \( h \). Displacement field is described by three linear displacements \( u_\alpha, w \) of middle surface and three rotations \( \phi_\alpha, \psi \). The following equations are to be valid (see [2] for details):

- geometrical relations (\( \gamma_{\alpha \beta}, \kappa_{\alpha \beta}, \gamma_{\alpha \lambda}, \kappa_{\alpha \lambda}, \gamma_{33} \) - strain components, \( e_{\alpha \beta} \) - Ricci symbol):
  \[ \gamma_{\alpha \beta} = u_\alpha u_\beta - e_{\alpha \beta} \psi; \quad \kappa_{\alpha \beta} = \phi_\alpha \phi_\beta, \quad \gamma_{33} = \phi_\alpha + w_\alpha, \quad \kappa_{33} = \psi_\alpha, \quad \gamma_{33} = \psi, \] (1)

- constitutive equations (\( N_{\alpha \beta}, M_{\alpha \beta}, N_{\alpha 3}, M_{\alpha 3} \) - internal forces, \( k, l \) – correction factors):
  \[ N_{\alpha \beta} = B^0_{\alpha \beta \mu} \gamma_{\mu \lambda}, \quad M_{\alpha \beta} = \frac{h^2}{12} B^0_{\alpha \beta \mu} \kappa_{\mu \lambda}, \quad N_{\alpha 3} = k^2 B^0_{\alpha 3 \beta 3} \gamma_{33}, \quad M_{\alpha 3} = \frac{h^2}{12} l^2 B^0_{\alpha 3 \beta 3} \kappa_{33}, \] (2)

- equilibrium equations (\( f_\beta, f_3, m_\beta, m_3 \) - external loads)
  \[ N_{\alpha \beta} + f_\beta = 0, \quad N_{\alpha 3} + f_3 = 0, \quad M_{\alpha \beta} - N_{\beta 3} + m_\beta = 0, \quad M_{\alpha 3} + e_{\alpha \beta} N_{\alpha \beta} + m_3 = 0. \] (3)

Constitutive equations for tensegrity plate-like orthotropic plates are discussed below.

3. Constitutive relations for orthotropic tensegrity plate-like structure

An example of orthotropic tensegrity plate-like structure is a system of dully connected repetitive expanded octahedron modules [5,6] – Fig. 1. It is assumed that the dimensions of the
repetitive module $a=h$. Struts, regular cables and connecting cables are described by Young modulus $E$ and cross sections $A$ with relations (5) with the self-stress (assumed even for each module) multiplied by $S$. The distances between three orthogonal pairs of struts (Fig. 1a) are uneven and defined as follows: $\alpha_1 = \frac{k}{K} = 0.65, \alpha_2 = \frac{l}{L} = 0.30, \alpha_3 = \frac{m}{M} = 0.56$.

![Fig. 1. Tensegrity module (expanded octahedron) and tensegrity plate.](image)

After the procedure described in the Introduction (see also [3,1] for details) non zero coefficients of the elasticity tensor of the tensegrity plate-like structure are the following:

$$B_{111}^0 = \frac{2EA}{h}(1 + 1.52325 \cdot k + 0.13125 \cdot m + 0.129225 \cdot \sigma),$$

$$B_{222}^0 = \frac{2EA}{h}(1 + 1.35912 \cdot k + 0.35 \cdot m + 0.137028 \cdot \sigma),$$

$$B_{112}^0 = 2B_{121}^0 = 2B_{122}^0 = \frac{EA}{h}(0.845615 \cdot k - 0.105243 \cdot \sigma),$$

$$B_{232}^0 = \frac{EA}{h}(1.51283 \cdot k - 0.168813 \cdot \sigma), \quad B_{313}^0 = \frac{EA}{h}(1.26604 \cdot k - 0.153207 \cdot \sigma).$$

(4)

where:

$$m = \frac{(EA)_{\text{connection}}}{(EA)_{\text{strut}}}, \quad k = \frac{(EA)_{\text{cable}}}{(EA)_{\text{strut}}}, \quad (EA)_{\text{strut}} = EA, \quad \sigma = \frac{S}{EA}. \quad (5)$$

The coefficients, and in consequence displacements, strains and internal forces depend on the proportions (5) and the level of self-stress. Some examples will be presented during the Conference for membrane and bending analysis of plate strips and simply supported rectangular plates. Orthotropic configurations of other tensegrity modules (3 and 4-strut Simplexes) will be also presented with discussion of results.

5. References


