THE CONTACT PROBLEM FOR PIECEWISE-HOMOGENEOUS ELASTIC PLATE REINFORCED BY FINITE ELASTIC STRINGER OF VARIABLE STIFFNESS

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1. Introduction

Considered problem refers to a wide class of contact and mixed value problems of elasticity theory. There are many problems considered early for various domains reinforced by a elastic stringers or thin inclusions, as well by a stringers of variable stiffness, for which were obtained as an exact, as well an approximate solutions. Particularly in [1] an effective solution of a contact problem for piecewise-homogeneous orthotropic plane with finite inclusion of variable stiffness changing by linear law was obtained.

In present work a analogue of problem [1] for piecewise-homogeneous isotropic plate assuming that stringer stiffness varies according to power law is considered. Solution of a problem is reduced to solution of a Prandtl’s singular integrodifferential equation with generalized Cauchy kernel. Depending on the exponents of stiffness variety law the weight functions describing the behavior of a solution in the vicinity of a stringer ends are found. Further, equation is solved by the method of mechanical quadratures, which developed in [2] for singular integral equations with generalized Cauchy kernel and in [3] for Prandtl’s singular integral.

2. Statement of problem and governing equation

A piecewise-homogeneous isotropic plate consist from two dissimilar semi-infinite plates is considered. One from semi-infinite plates is stiffened by thin stringer of length \( l \), which is perpendicular to and terminating at bimaterial interface, as well have a width varying by power law \( h(x) = h_0 x^p (1-x)^q \quad (p, q \geq 0) \). It is supposed that stringer fastened with plate, don’t resist to buckling and stretched or compressed as the rod being in a state of uniaxial stress.

Equating stringers axial deformation under an applied external load thereto \( q_0(x) \) and unknown contact stresses \( \tau(x) \) with the plate deformations in the contact area from the same contact stress \( \tau(x) \) obtain governing equation of stated problem. In dimensionless values it written as:

\[
\int_{-1}^{1} \left( \frac{1}{\xi - \xi} + \frac{B}{\xi + \xi^2} \right) \varphi(\xi)d\xi = A(\zeta) \left[ \int_{-1}^{\zeta} \varphi(\xi)d\xi - \int_{-1}^{\zeta} q_0(\xi)d\xi \right], \quad (-1 < \zeta < 1)
\]

where

\[
\varphi(\xi) = \frac{I}{T} \tau(t); \quad T = \int_{-1}^{1} q_0(\xi)d\xi; \quad A(\zeta) = A_0 (1 + \zeta)^{-\rho} (1 - \zeta)^{-\eta}; \quad A_0 = \frac{2^{1+\rho+\eta} \pi \mu_i \left( \frac{1 - \nu_0^2}{E_0 h_0} \right)}{
\text{B} \quad \text{C} \quad \text{are constants depending at Poisson ratios of semi-infinite plate materials and ratio of their shear modulus, } \nu_0 \text{ are elasticity modulus and Poisson ratio of a stringer material, } \mu_i \text{ is a shear modulus of semi-infinite plate where stringer is located.}
}
3. Behavior of a solution in the neighbourhood of a stringer ends

The investigation of a equation behavior near the ends of interval of integration is shown that behavior of solution near the the ends is strong depend from exponents $p$ and $q$.

When

1) $0 < p < 1$ and $0 < q < 1$ we have

$$
\varphi(\zeta) = \varphi^*(\zeta)(1-\zeta)^{1/2} (1+\zeta)^{-\alpha} + \psi^*(\zeta)(1-\zeta)^{1/2} (1+\zeta)^{1-p-\alpha}
$$

$0 < \alpha < 1$ is a root of $\cos \pi \alpha + B - \alpha C = 0$, otherwise $\alpha = 0$

2) $0 < p < 1$ and $q = 1$

$$
\varphi(\zeta) = \varphi^*(\zeta)(1-\zeta)^{1/2} (1+\zeta)^{-\alpha} + \psi^*(\zeta)(1-\zeta)^{1/2} (1+\zeta)^{1-p-\alpha}
$$

$0 < \gamma \leq 0.5$ is a root of $\pi \cot \gamma \frac{2A_0}{1-\gamma} = 0$ and $0.5 \leq \delta < 1$ is a root of $\pi \cot \delta \frac{2A_0}{1+\delta} = 0$

3) $0 < p < 1$ and $q > 1$

$$
\varphi(\zeta) = \varphi^*(\zeta)(1-\zeta)^{q-1} (1+\zeta)^{-\alpha} + \psi^*(\zeta)(1-\zeta)^{q-1} (1+\zeta)^{1-p-\alpha}
$$

4) $p = 1$ and $0 < q < 1$

$$
\varphi(\zeta) = \varphi^*(\zeta)(1-\zeta)^{1/2} (1+\zeta)^{-\eta} + \psi^*(\zeta)(1-\zeta)^{1/2} (1+\zeta)^{0}
$$

$0 < \eta < 1$ is a root of $\pi \cot \eta \frac{\pi}{\sin \eta} B - \frac{\eta \pi}{\sin \eta} C = \frac{2A_0}{1-\eta}$

$0 < \theta < 1$ is a root of $\pi \cot \theta \frac{\pi}{\sin \theta} B + \frac{\eta \pi}{\sin \theta} C = -\frac{2A_0}{1+\theta}$

5) $p = 1$ and $q = 1$

$$
\varphi(\zeta) = \varphi^*(\zeta)(1-\zeta)^{1/2} (1+\zeta)^{0} + \psi^*(\zeta)(1-\zeta)^{0} (1+\zeta)^{0}
$$

6) $p = 1$ and $q > 1$

$$
\varphi(\zeta) = \varphi^*(\zeta)(1-\zeta)^{q-1} (1+\zeta)^{-\eta} + \psi^*(\zeta)(1-\zeta)^{q-1} (1+\zeta)^{0}
$$

7) $p > 1$ and $0 < q < 1$

$$
\varphi(\zeta) = \varphi^*(\zeta)(1-\zeta)^{1/2} (1+\zeta)^{p-1} + \psi^*(\zeta)(1-\zeta)^{1/2} (1+\zeta)^{p-1}
$$

8) $p > 1$ and $q = 1$

$$
\varphi(\zeta) = \varphi^*(\zeta)(1-\zeta)^{1/2} (1+\zeta)^{p-1} + \psi^*(\zeta)(1-\zeta)^{1/2} (1+\zeta)^{p-1}
$$

9) $p > 1$ and $q > 1$

$$
\varphi(\zeta) = \varphi^*(\zeta)(1-\zeta)^{q-1} (1+\zeta)^{p-1}
$$

The new unknown functions $\varphi^*(\zeta)$ and $\psi^*(\zeta)$, which are smooth functions bounded on closed interval $[-1,1]$, will be found by the method of mechanical quadratures.

4. References

