**SELF-EQUILIBRIUM GEOMETRY OF THE CLASS-THETA TETRAHEDRAL TENSEGRITY MODULE**

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## 1. What is the *Class k* and *Class Θ* tensegrity system?

An arrangement of rigid bodies forms a tensegrity configuration if the given arrangement can be stabilized by some set of tensile members connected between the rigid bodies. The tensegrity system is composed of a tensegrity configuration of rigid bodies, usually the bars, and any given set of tensile members, usually the cables, connecting the rigid bodies [1]. Examples are shown in Fig.1a-d. Strictly speaking Fig. 1d shows a stiff regular minimal tensegrity prism, because the minimal number of cables that can stabilize of this system is 9. A similar but non-minimal system, for comparison depicted in Fig. 1e, has 12 cables. For instance a regular minimal 3-bar tensegrity prism, see Fig. 1d, is regular because tops and bottoms have same vertical centerline and are parallel. As such, tensegrity systems and structures differ from regular trusses by purposefully designing all tensile elements to be cables. The result is a lightweight structure with comparable stiffness properties to regular truss structures.

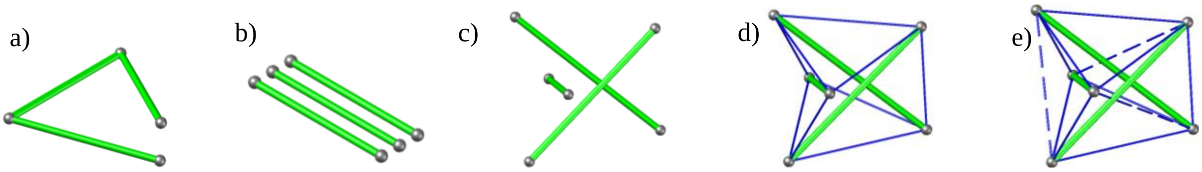


Fig. 1. A tensegrity configuration and tensegrity system: a) and b) not a tensegrity configuration, c) the tensegrity configuration, d) the minimal tensegrity system, e) the non-minimal tensegrity system.

The *Class* ***k*** tensegrity system, according to the strictest definitions, is a structural unit based on the use of isolated components in compression inside a net of continuous tension. The tensegrity systems depicted in Figs 1d, 1e and 2a is said to be of the *Class* ***k*** = 1, since each node is connected to one compressive member only [1]. Connections between compressive elements are achieved by flexible tensile cable elements, on the assumption that all nodes are the frictionless hinged joints. In this way, the compressive members do not experience a bending moment. Next examples are shown in Figs 2d and 2e.

The *Class* ***Θ*** tensegrity is a self-equilibrated system composed of discontinuous set of compressed components and the separate net of cables located together inside the other cable network, therefore a net of tension is discontinuous [2].

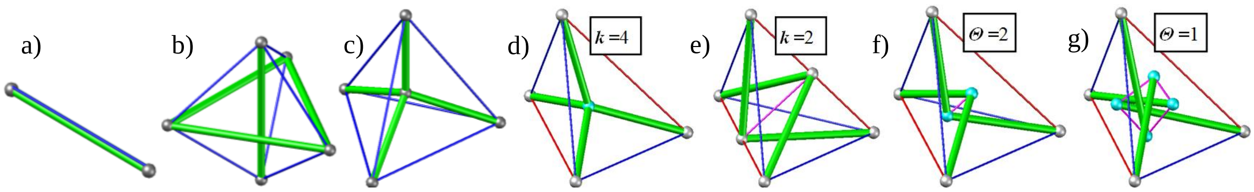


Fig. 2. Some examples of the Class ***k*** and *Class* ***Θ*** tensegrity systems: a) Class ***k***=1, b) Class ***k***=2, c) Class ***k***=4. Possible topologies of the tensegrity tetrahedron: d) by Fuller (*Class* ***k*** =4), e) by Motro (*Class* ***k*** =2), f) the *Class* ***Θ*** =2 tetrahedral cell, g) the *Class* ***Θ*** =1 tetrahedral cell.

## 2. Mathematical model of the *Class Θ* = 1 tetrahedral tensegrity module

The following is a mathematical model for figures related to the *Class Θ* = 1 tensegrity tetrahedron, explaining why the tensegrity module is a stable construction, albeit with infinitesimal mobility. Consider a space-filling tetrahedron of one edge length *2l* centred at the origin of x axis, and its second edge length *2l* which is parallel to the y axis of Cartesian system as shown in Fig. 3. If the Cartesian coordinates of one edge or cable are (- *l* , 0 , 0) and (*l* , 0 , 0), then its second edge or cable coordinates will be respectively (0 , -*l* , *l*) and (0 , *l* , *l*). The same is true of four bars. The coordinates of the other bar ends are also recorded.

Imagine this tensegrity module built from bars of given length *b* and internal cables of given length *c* connecting neighbouring bar ends. The theoretical relation tells us there are two extreme positions for α: one realized by putting the bars apart as shown in Fig. 3c, the other by pushing them together as shown in Fig. 3d. The direction of rotation concerning the cable from *TB2* to *TB1* is depicted with an arrow and is equal to zero at position according to Fig. 3c. In the particular case, between the first and second positions, the Fig. 3a-b is the stable tensegrity tetrahedron. Since the tensegrity tetrahedron represents an extremal point of the relation recorded in (1), it has infinitesimal mobility.

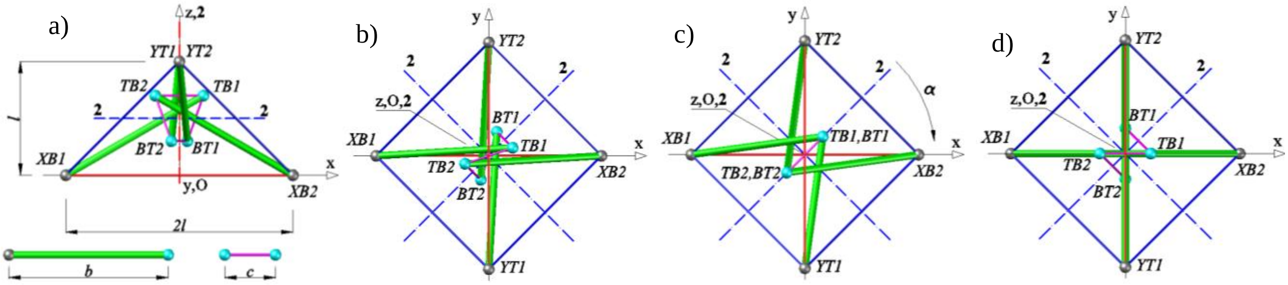


Fig. 3. Model of the *Class Θ* =1 tensegrity tetrahedron: a) front view of the mother configuration, b) top view of the mother configuration, c) top view of model in the first extreme position for α=0°, d) top view of model in the second extreme position for α=45°.

The value *b*, that is the length each of four bars, can be determined by subtracting the coordinates of one bar end (e.g. *XB1*) from coordinates of the other end (*TB1*). Therefore, the length of bar between *TB1* and *XB1*:

$$b = \left[\frac{1}{4}c^2 + l^2 + \frac{1}{4}(l + c\cos\alpha)^2 + l c\cos\left(\frac{\pi}{4} - \alpha\right)\right]^{\frac{1}{2}} \quad (1)$$

Thus the relation (1) is the function of several variables: *l*, *c* and α. Therefore, if the set of values *l*, *c* and α will be properly established, there are only two stable configurations of a tetrahedral tensegrity system: positive and negative twist. In the circumstances the length of bars will be maximal extended. Only when the distances between *TB1* and *XB1*, *TB2* and *XB2* etc. achieve a maximum, then all the elements are stressed and the tensegrity system can be stable. The study will be concern the results of the relation (1). An original approach introduced to tensegrity design greatly simplifies the form finding problem independently of material choices and external loads.

## 3. References


[1] Skelton R.E., Helton J.W., Adhikari R., Pinaud J.P., Chan W. (2002) *An Introduction to the Mechanics of Tensegrity Structures, Dynamics and Control of Aerospace Systems*, University of California, San Diego, CRC Press LLC

[2] Bieniek Z. (2015) Examples of Cable-Bar Modular Structures Based on the Class-Theta Tensegrity Systems, *Journal of Civil Engineering and Architecture*, **9**, 1452-1462