TWO-PHASE ISOTROPIC COMPOSITES OF EXTREMAL MODULI. THE INVERSE HOMOGENIZATION PROBLEM

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1. Motivation. Short description of the problem.

The present work deals with the inverse homogenization problem: to reconstruct the layout of two well-ordered elastic and isotropic materials characterized by the bulk and shear moduli: (κ_1, μ_1), (κ_2, μ_2) within a 2D periodicity cell Y, corresponding to the predefined values of the moduli (κ^*, μ^*) of the effective isotropic composite and to the given isoperimetric condition concerning the volume fraction ρ of the 2nd material. The algorithm used follows from imposing the finite element approximation on the solution to the basic cell problems of the homogenization theory. The periodicity cell Y is uniformly divided into finite elements each of different element-constant wise (also non-isotropic) material.

In the planar problems the theoretically admissible pairs (κ^*, μ^*), for given ρ lie within a rectangular domain of vertices determined by the Hashin-Shtrikman bounds. The tightest bounds (called further CG bounds) known up till now are due to Cherkaev and Gibiansky [1]. They form a curvilinear rectangle. The microstructures corresponding to the interior of the CG rectangle can be of arbitrary rank, in the meaning of the hierarchical homogenization. Moreover, it is well known that the rank-1 microstructures cannot attain the regions close to the vertices and boundary lines of the CG curvilinear rectangle. The specific challenge is to build such ”extremal” [8] composites that reach points lying on the boundaries of the CG area. In the present work, in order to achieve the boundary of the CG rectangle, a variety of isotropic composites are constructed based on different underlying microstructures.

The SIMP-constructed materials, single-scale laminates (similar to the Kagome lattices) and 2nd rank orthogonal laminates as an underlying microstructure are obtained and will be presented. For the 1st rank composites (constructed by SIMP), the defined in this way the problem of the inverse homogenization is a difficult, binary integer programming problem with a large number of variables. The relaxed formulation admits mixing the given materials in some proportion with the material properties of intermediate density values (e.g. RAMP [9], GRAMP [3]) and the one proposed by the present author: HSρ, see [7]). Here, the parameters of density are the only decision variables in each finite element. For the underlying laminates the available exact equation [6] is used to determine homogenized constitutive values described by the density and additional parameters (for each element) treated together as decision variables in the optimization process. In the present work the material properties of each element can be described by 1 to 3 independent parameters. The number of parameters depends on the selected type of the underlying structure.

The inverse problem thus formulated, although strongly nonlinear, can be solved numerically by the gradient method with using of some additional techniques [5] (e.g. multiple starting points, a filtering of the gradient, adequate penalization of the objective function etc.). One way to solve such nonlinear optimization problem is its replacing by a sequence of linearized problems [4]. In the presented solution the problem, for all utilized types of underlying structures, is effectively solved numerically by the Sequential Linear Programming (SLP) method. The effective isotropic moduli for the analyzed microstructure as well as the gradient are computed according to the homogenization algorithm using FE techniques along with periodicity assumptions.

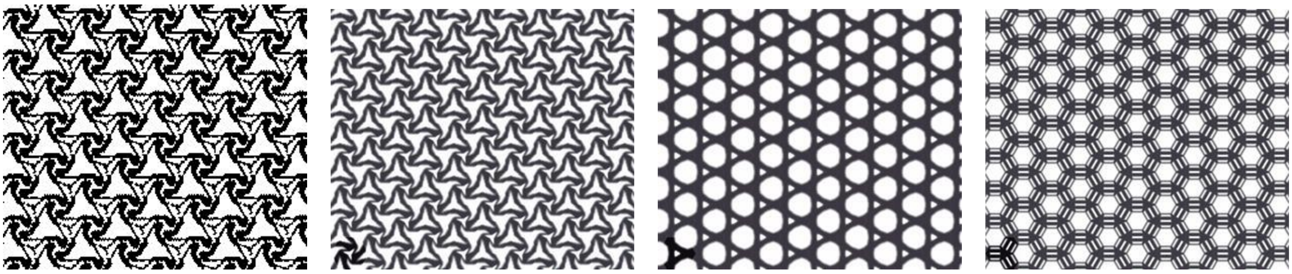

Figure 1. Examples of isotropic two-phase “extremal” structures of 1st rank obtained using the proposed method.

The present work is limited only to the case of the isotropy of the homogenized material. Usually, for the inverse homogenization task the shape of the periodic cells is chosen arbitrarily, often as a square or a rectangle. This choice means necessity of introduction additional constraints to the formulated problem, i.e. “isotropy constraints”. The key point of presented algorithm is the use of the hexagonal cell of periodicity with assumed rotational symmetry of angle of 120 degrees. Such a cell shape ensures exact isotropy of periodic composites for any constituents’ materials (including voids, orthotropic, anisotropic). Adopting such a cell shape one can essentially reduce the number of constraints involved in the optimization problem and moreover it results (due to symmetry) in a significant reduction in the number of design variables to the optimization problem considered. It is worth emphasizing that the results obtained lie fairly close to the assumed ones located on the CG bound.

The present paper delivers a method of endowing the isotropic material designs (by the IMD, proposed recently by Czarnecki [2]) with microstructures reflecting the optimal isotropic properties.

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2. References

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