EFFICIENT ALGORITHMIC TREATMENT OF THE INCREMENTAL MORI–TANAKA SCHEME FOR ELASTO-PLASTIC COMPOSITES

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1. Introduction

A consistent algorithmic treatment of the incremental Mori–Tanaka model for elasto-plastic two-phase composites is developed. The computational scheme can be classified as a doubly-nested iteration-subiteration scheme. At each level, the corresponding system of nonlinear equations is solved using the Newton method. Exact linearization is performed at each level so that quadratic convergence of the nested iterative scheme is achieved. The model provides reliable results which are comparable to those known from the literature. The efficiency of the numerical code has been tested for large-scale finite-element problems. The convergence behaviour has been found similar to that of the simple Huber-von Mises plasticity model.

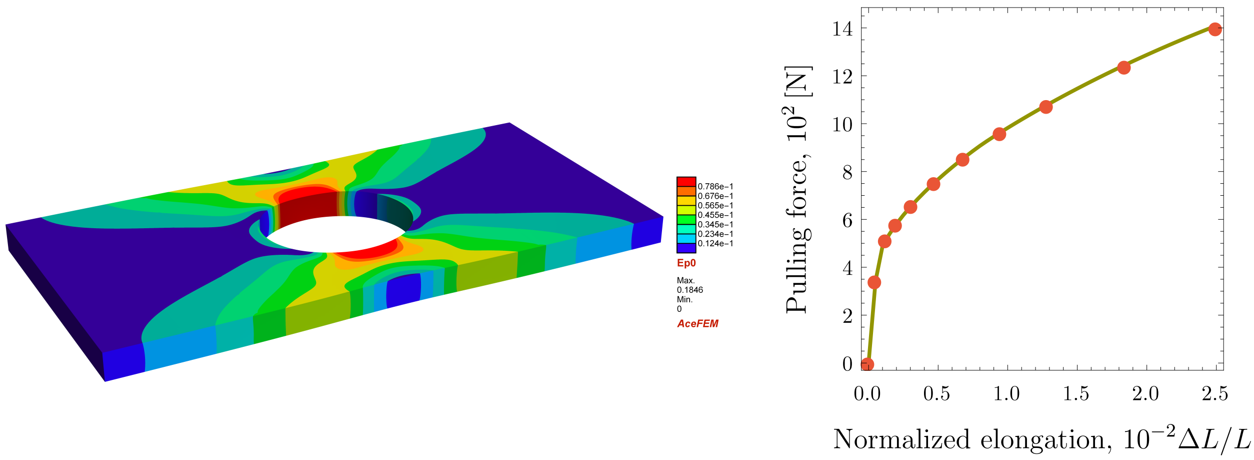
2. Implementation of the incremental Mori–Tanaka scheme

The Mori–Tanaka model [1] is a mean-field model originally dedicated to the estimation of the effective properties of linearly elastic two-phase composites. It belongs to the family of models based on the Eshelby's solution to the problem of an ellipsoidal inclusion embedded into an infinite linearly elastic matrix [2]. By applying linearization of the constitutive law of each phase, as proposed by Hill [3], the original MT model can be adapted to elasto-plastic deformation of composites. Nowadays, this theory is well established. However, its *efficient* practical utilization in large-scale boundary value problems seems to be still problematic. The pioneering work in this field is by Doghri and Ouaar [4] but still their algorithm do not lead to a consistent global tangent matrix.

The proposed numerical implementation provides comprehensive treatment of the MT scheme. This has not been done so far to the author's best knowledge and is the main purpose of our work. As stated in the introduction, our scheme consists of three nested iterative schemes. The lowest level is constituted by independent sets of incremental constitutive equations of each phase. They are solved using the return mapping algorithm and provide an update of internal variables for a given (yet unknown) increment of the strain in the individual phase. Additionally, the first and the second derivative of internal variables with respect to independent variables (strain in phases) is computed. Those derivatives are crucial for obtaining a consistent global tangent matrix. At the intermediate level, the interaction equation [3], appropriately formulated for the incremental scheme, is solved. It delivers proper redistribution of strains and stresses among the phases according to the Mori–Tanaka model for the given overall strain in the composite. At the outer level, the global equilibrium equations are solved. Exact linearization is performed at each level of the nested scheme. As a result, the global tangent matrix is an exact linearization of the governing equations, and quadratic convergence of the Newton method can be achieved.

3. Finite-element example

Finite-element implementation and computations have been performed with the use of the *AceGen/AceFEM* system [5]. In particular, the exact algorithmic (consistent) tangent has been obtained by applying the automatic differentiation (AD) technique available in the *AceGen* system. As a result, large-scale simulations can be efficiently performed with use of the proposed implementation.



(a) (b)

Figure 1. Uniaxial tension test of the slab with a hole: (a) equivalent plastic strain in the matrix and (b) pulling force as a function of normalized elongation. Solid line corresponds to constant load increment $\Delta L/L =$ 0.0005 (50 steps), while dots indicate the results obtained with maximal possible increment of load (10 steps).

As an example, consider a thin plate with a hole made of a metal matrix composite. The content of spherical ceramic inclusions, uniformly dispersed in the aluminum alloy matrix, amounts to 20% of volume fraction. The plate is stretched in the longitudal direction. The inclusions are assumed elastic, and the matrix is elasto-plastic, governed by J_2-plasticity with isotropic hardening. Distribution of the equivalent plastic strain in the matrix is shown in Fig. 1(a). The finite-element mesh comprises 368 640 trilinear hexahedral elements which corresponds to approximately $1.2 \cdot 10^6$ DOF. The pulling force as a function of normalized elongation is shown in Fig. 1(b). The figure presents results of two analyses: the solid line corresponds to a small, constant load increment, while the dots refer to the solution proceeding with possibly large load increments. The convergence behaviour is similar to that of the simple J_2-plasticity model, which shows that the *proposed* numerical implementation of the micromechanical model can serve as an *extremely efficient* tool for solving large-scale FE problems dedicated to elasto-plastic composite materials.

4. References

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