PARAMETER-FREE SHAPE-SIZE OPTIMIZATION FOR DEFORMATION TAILORING OF A FRAME STRUCTURE

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1. Background and objective

Because of their lightweight and eco-friendly features, Frame structures composed of straight or curved members ranging over multiple scales have been widely utilized in many engineering applications such as stadiums, bridges, automobile bodies, micro electro mechanical system (MEMS) parts. Despite the fact that a frame structure has the appearance of being assembled simply by slender members, an effectively designed frame structure can perform with excellent mechanical characteristics such as high load-carrying capacity and high natural frequency while an inadequate design may cause problems including insufficient stiffness, self-excitation, buckling, and so on. Optimization design has been one of the essential techniques for designing high performance structures economically. Thus, a smart and high performance frame structure for a specific function can be designed by applying such optimization techniques. For tailoring the stiffness or achieving the desired deformation of frame structures against external loads, the present work proposes a parameter-free shape-size optimization method, a free form-size design method for frame structures without any shape-size parameterization, in which both shape variation and size variation of cross-sectional area $A(\boldsymbol{x})$ of a frame structure are treated as design variable functions.

2. Problem formulation

Figure 1 shows the schematic diagram of domain variation for designing a frame structure consisting of infinitesimal straight Timoshenko beam elements with arbitrary cross section, where shape optimization and size optimization are implemented simultaneously. In shape optimization, we assume that a beam j with an initial domain Ω^j and a centroidal axis S^j undergoes shape variation $\boldsymbol{V}(\boldsymbol{x})$ in the off-axis direction to the centroidal axis such that its domain and centroidal axis become Ω_s^j and S_s^j, respectively [1]. The subscript s expresses the iteration history of the domain variation, or the design time. Define $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$ as the unit vectors of the centroidal axis in the x_1 and x_2 directions, respectively, $\boldsymbol{V}^j(x)$ can be expressed by $\boldsymbol{V}_1^j$ and $\boldsymbol{V}_2^j$ as:

(1) $$\boldsymbol{V}^j(x) = \boldsymbol{V}_1^j(x) + \boldsymbol{V}_2^j(x) = \left(\boldsymbol{V}^j \cdot \boldsymbol{n}_1^j\right)\boldsymbol{n}_1^j + \left(\boldsymbol{V}^j \cdot \boldsymbol{n}_2^j\right)\boldsymbol{n}_2^j$$

In size optimization, we consider that a beam has an initial cross-sectional area $A^j(x)$, and that its cross-sectional area becomes $A_s^j = A^j + \delta A^j$ according to the size variation.

Achieving a desired static deformation of a frame structure, we introduce the sum of squared error norms for the desired displacements on specified members as the objective functional and the assumption that each frame member varies in the off-axis direction with changing cross sections. Considering the state equation for a frame structure and the volume as the constraint conditions, a distributed-parameter shape-size optimization problem for finding the optimal shape and size distributions is formulated in the function space.

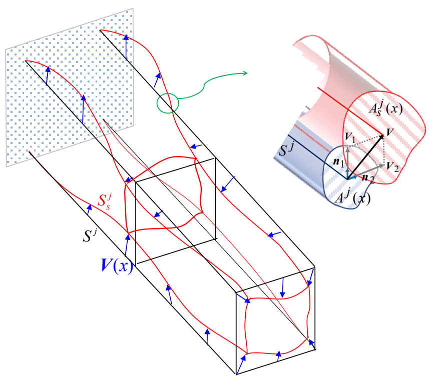

Figure 1. Shape and size variation.

The shape gradient functions (i.e. sensitivity function for shape variation) $G_1\boldsymbol{n}_1(x)$, $G_2\boldsymbol{n}_2(x)$ with respect to the shape variation in the $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$ directions [1], the size gradient function $G_A(x)$, and the optimality conditions for this problem are theoretically derived with the Lagrange multiplier method, the material derivative method and the adjoint variable method. The optimal shape-size variations $\boldsymbol{V}(x)$, $\delta A(x)$ that minimize the objective functional are determined by using the H^1 gradient method for frame structures.

3. H^1 gradient method for determining shape and size variations

In our previous work [1], the H^1 gradient method for frame structures, a gradient method in the function space with mesh regularization, was developed. In this work, the method is expanded for shape-size simultaneous optimization of frame structures. The negative shape gradient functions $-G_1\boldsymbol{n}_1(x)$, $-G_2\boldsymbol{n}_2(x)$ are applied as distributed forces to the fictitious elastic frame structure under the Robin condition as shown in Fig. 2 (a). The optimal shape variation $\boldsymbol{V}(x)$ is determined as the displacement field in this fictitious analysis. The negative size gradient function $-G_A(x)$ is applied as a distributed heat supply per unit volume to the fictitious elastic frame structure under the Robin condition as shown in Fig. 2 (b). The optimal size variation $\delta A(x)$ is determined as the temperature field in this fictitious heat transfer analysis. The obtained $\boldsymbol{V}$ and δA are used to update the shape and the cross-sectional area. It has been verified that this method, a gradient method reduces the objective functional [1].

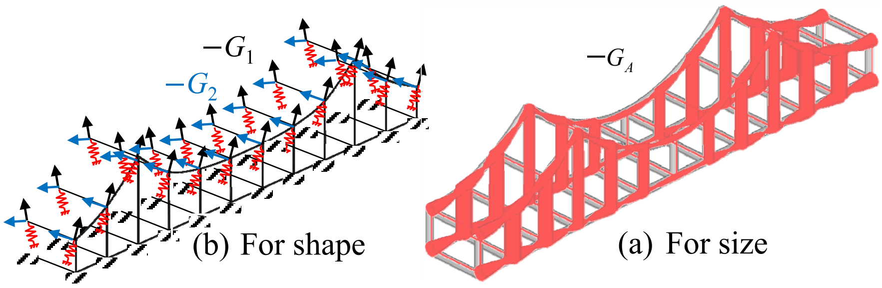

Figure 2. H^1 gradient method for frame structures.

4. Numerical result

The proposed method is applied to a switch model. The target displacement is set as $+2\|\boldsymbol{w}_h\|$ in the X_1 direction at the controlled point on the right end under the enforced displacement $-\|\boldsymbol{w}_h\|$ to the left end. Figures 3 (a) and (b) show the states of the initial and the optimized design, respectively. The grey and black curves express the states before deformation and after deformation, respectively. It is confirmed that the tailored deformation is surely achieved with this method.

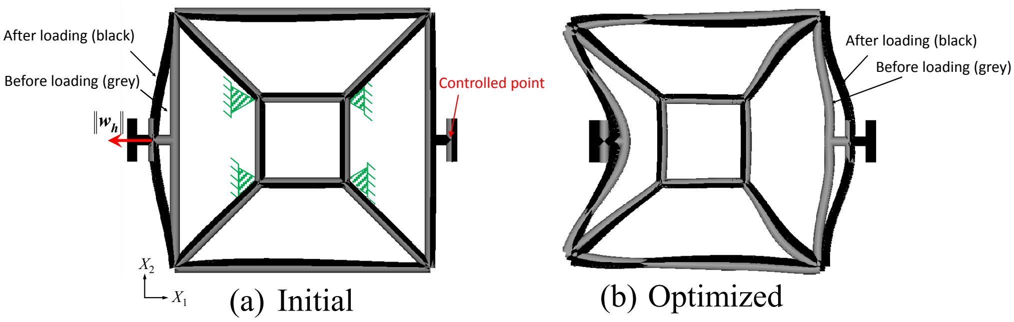

Figure 3. Calculated result of switch model.

5. References

[1] M. Shimoda, Y. Liu and T. Morimoto (2014), Non-parametric free-form optimization method for frame structures. *Struct. Multidisc. Optim.*, **50** (1), 129-146.