

# ANALYSIS OF THERMAL PROCESSES OCCURING IN THE HEATED MULTILAYERED METAL FILMS USING THE DUAL-PHASE LAG MODEL

*E. Majchrzak and G. Kałuza*

*Silesian University of Technology, Gliwice, Poland*

## 1. Introduction

Multilayered thin metal film subjected to the ultra-short laser pulse is considered. Mathematical description of the process discussed is based on the system of the dual-phase lag equations supplemented by appropriate boundary and initial conditions [1, 2, 3]. Special attention is devoted to the ideal contact conditions at the interfaces between the layers, which in the case of the dual-phase lag model must be formulated in a different way than in the macroscopic Fourier model [4]. To solve the problem the explicit scheme of finite difference method is developed. In the final part the example of computations is shown.

## 2. Governing equations

Multilayered thin film of thickness  $L = L_1 + L_2 + \dots + L_M$  with an initial temperature distribution  $T(x, 0) = T_0$ , constant thermal properties of successive layers and the ideal thermal contact between the layers is considered. The temperature distribution in the successive layers is described by the system of equations (1D problem) [1, 3]

$$(1) \quad L_{m-1} < x < L_m : \quad c_m \left[ \frac{\partial T_m(x, t)}{\partial t} + \tau_{qm} \frac{\partial^2 T_m(x, t)}{\partial t^2} \right] = \lambda_m \frac{\partial^2 T_m(x, t)}{\partial x^2} + \tau_{Tm} \lambda_m \frac{\partial^3 T_m(x, t)}{\partial t \partial x^2} + Q_m(x, t) + \tau_{qm} \frac{\partial Q_m(x, t)}{\partial t}, \quad m = 1, 2, \dots, M$$

where  $c_m$  is the volumetric specific heat of  $m$ -th layer,  $\lambda_m$  is the thermal conductivity,  $\tau_{qm}$  is the relaxation time,  $\tau_{Tm}$  is the thermalization time,  $T_m$  is the temperature,  $x$  is the spatial co-ordinate and  $t$  is the time. A front surface  $x = 0$  is irradiated by a laser pulse and the source function  $Q_1(x, t)$  connected with the laser heating is defined as follows [2]

$$(2) \quad Q_1(x, t) = \sqrt{\frac{\beta}{\pi}} \frac{1-R}{t_p \delta} I_0 \exp \left[ -\frac{x}{\delta} - \beta \frac{(t-2t_p)^2}{t_p^2} \right]$$

where  $I_0$  is the laser intensity,  $t_p$  is the characteristic time of a laser pulse,  $\delta$  is the absorption depth,  $R$  is the reflectivity of the irradiated surface and  $\beta = 4 \ln 2$ . For  $m = 2, 3, \dots, M$ :  $Q_m(x, t) = 0$ . For  $x = 0$  and  $x = L$  the non-flux conditions can be assumed.

The boundary conditions on the contact surfaces between sub-domains have the form of continuity ones, this means

$$(3) \quad x = L_m : \quad \begin{cases} T_m(x, t) = T_{m+1}(x, t) \\ q_m(x, t) = q_{m+1}(x, t) \end{cases}, \quad m = 1, 2, \dots, M-1$$

The initial conditions are also given

$$(4) \quad t = 0: \quad T_m(x, 0) = T_{m0}, \quad \left. \frac{\partial T_m(x, t)}{\partial t} \right|_{t=0} = 0$$

It should be pointed out that in the dual-phase lag model the following relation between heat flux and temperature gradient must be apply [1]

$$(5) \quad q_m(x, t + \tau_{qm}) = -\lambda_m \frac{\partial T_m(x, t + \tau_{Tm})}{\partial x}$$

The dependence (5) should be included in the boundary conditions (3).

### 3. Example of computations

The layer of thickness  $L = 100$  nm being a composition of gold layer  $L_1 = 50$  nm and chromium layer  $L_2 = 50$  nm subjected to a short-pulse laser heating ( $R = 0.93$ ,  $I_0 = 30$  J/m<sup>2</sup>,  $t_p = 0.1$  ps,  $\delta = 15.3$  nm) is considered.

The values of thermal parameters are taken from [3]. The problem is solved using the explicit scheme of finite difference method under the assumption that grid step is equal to  $h = 1$  nm and the time step is equal to  $\Delta t = 0.0002$  ps.

In Figure 1 the temperature distribution for time 0.5 ps is shown. The first curve corresponds to the solution in which the boundary condition is formulated taking into account the formula (5), while second curve is obtained under the assumption that the classical continuity condition is applied.

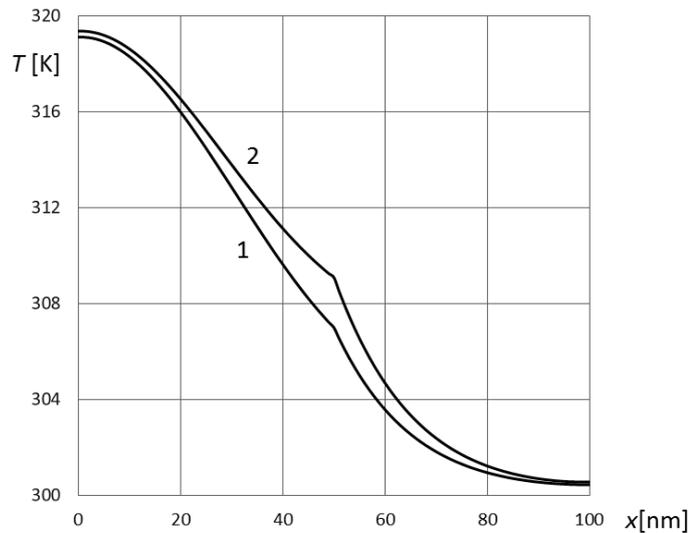


Figure 1. Temperature distribution 1- DPL, 2 - DPL with the classical continuity condition.

### 4. Conclusion

In considerable amount of works concerning the dual-phase lag equation modeling in the case of multilayered domains the system of PDE is supplemented by the continuity conditions in the classical macroscopic form. Such an approach is incorrect, of course, and the acceptance of the 'microscale' form of this condition gives the results evidently different than in the case of simplified model.

### 5. References

- [1] D.Y.Tzou (1997). *Macro- to Microscale Heat Transfer. The lagging behaviour*, Taylor and Francis.
- [2] J.K.Chen and J.E.Beraun (2001). Numerical study of ultrashort laser pulse interactions with metal films, *Numer. Heat Tr A-Appl.*, **40**, 1-20.
- [3] E.Majchrzak, B.Mochnacki, A.L.Greer, J.S.Suchy (2009). Numerical modeling of short pulse laser interactions with multi-layered thin metal films, *CMES: Comp. Model. Eng.*, **41**, 2, 131-146.
- [4] J.R. Ho, Ch.P. Kuo, W.S. Jiaung (2003). Study of heat transfer in multilayered structure within the framework of dual-phase-lag heat conduction model using lattice Boltzmann method, *Int. J. Heat Mass Transfer*, **46**, 55-69.

This work is supported by the project BK-255/RMT4/2016.