FUNDAMENTALS OF THE THEORY OF SURFACE GROWTH WITH APPLICATIONS TO GEOMECHANICS AND AM TECHNOLOGIES

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1. Introduction

A vast majority of objects or solids surrounding us arise from some surface growth processes. Examples include many natural phenomena such as the growth of space objects, biological tissues, glaciers, blocks of sedimentary and volcanic rocks, etc. Similar processes determine the specific features of modern digital additive manufacturing technologies in industry, including well-known technologies such as stereolithography, electrolytic deposition, thermal and laser-based 3D printing, 3D-IC fabrication technologies, etc. Recent research has shown that solids formed by growth processes substantially differ in their properties from solids in the traditional sense of the word. Further, the classical approaches of solid mechanics fail when modeling the behavior of growing solids and should be replaced by new ideas and methods of modern mechanics, mathematics, physics, and engineering sciences.

In technology and nature, there are two main kinds of continuous growth processes known to researchers, namely, bulk and surface growth processes. Here we consider the latter, while for the former we refer the reader to the monographs [1, 2], where fundamental results concerning bulk growth are presented.

2. Basic concepts

We suggest an approach to modeling surface growth processes in solids on the basis of the following postulates:

- The surface growth of a solid is modeled by the motion of its boundary due to the influx of new material to the surface of the solid.
- The boundary conditions on the moving boundary (the growth surface) are found from an additional solid–surface contact interaction problem depending on the specific features of the growth process.
- The strain rate tensor (or the stretch rate tensor) of a growing solid is compatible, while the strain tensor is, as a rule, incompatible. Accordingly, it is absolutely natural to take the stress rate tensor, the strain rate tensor, and the velocity vector to be the main variables in the constitutive equations describing the surface growth process.

In general, a boundary value problem for a growing solid contains three dependent controlled groups of values, namely, the surface and bulk loads, the stresses on the accretion surfaces, and the material influx rate on the accretion surfaces. We have considered the case in which the velocities of the boundary particles of the growing solid are much smaller than the influx velocity of new particles, the strains are small, and the bulk force is zero, stated an appropriate boundary value problem (see also [3–4]), and proposed a method for its solution. Some results concerning applications of the theory to problems in geomechanics (e.g., see [5]) and AM technologies (e.g., see [6–8]) are presented below.

3. Applications

Mining engineers often use formulas derived for an elastic half-space under gravity to find the stress and strain values inside the Earth. We found this mathematical model to be inadequate in this situation, because the solution of the problem for a heavy half-space does not correspond to the asymptotic solution of any problem for an elastic ball under self-gravity. Then we considered the latter problem and discovered that its solution gives severe circumferential compressive stresses of the order of several tons per centimeter on the ball surface. This clearly contradicts our everyday experience and makes such a model of the Earth inadequate as well. We proposed a simple model of a growing elastic ball corresponding to the Earth accretion process. The closed-form solution of the ball growth problem shows the following interesting results. The stresses on the ball surface are zero. The solid ball is loaded like a liquid, and all interior stresses are equal to each other and do not depend on the elastic moduli. The shear stress intensity in such a growing ball is zero at any point. We also considered more complicated models of a growing ball taking into account the viscoelastic properties, nonuniformity, and aging of its material as well as the effect of rotation around an axis and stated qualitative conclusions.

As to the AM technologies, we studied models of growth processes for a vibrating plate, an arch structure, a shaft under torsion, etc. with applications to engineering and technology. A new holographic device for the identification of mechanical properties of growing solids was developed. The electrolytic deposition process was studied experimentally. The results of theoretical and experimental research demonstrate the unique properties of growing machine and instrument parts and structural members. It was found that considerable residual stresses, as well as noticeable shape distortion due to these stresses, are an inherent property of growing solids. The solution of the above-mentioned problems will ultimately permit, on the one hand, predicting the mechanical behavior of growing natural objects and, on the other hand, significantly improving the performance of AM fabricated parts and developing AM technologies themselves.

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4. References

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