A CIRCULAR INCLUSION WITH INHOMOGENEOUS ROUGH IMPERFECT INTERFACE IN HARMONIC MATERIALS

D.R. McArthur[1] **and L.J. Sudak**[1]
[1]*Department of Mechanical Engineering, University of Calgary, Alberta, Canada*

1. Introduction

In the following work, a rigorous study is presented for the problem associated with a circular inclusion embedded in an infinite harmonic matrix material in finite plane elasto-statics. The inclusion/matrix boundary is treated as a circumferentially inhomogeneous imperfect interface that is described by a linear spring-type imperfect interface model where in the tangential direction the interface parameter is infinite in magnitude and in the normal direction the interface parameter is finite in magnitude (the so called non-slip interface condition). Through the repeated use of the technique of analytic continuation the boundary value problem for four analytic functions is reduced to solving a single first order linear ordinary differential equation with variable coefficients for a single analytic function defined within the inclusion.

2. Formulation

We begin by defining the governing equations for a simply connected inclusion embedded in an infinite matrix both comprised of a type-1 harmonic material as described by [1], [2] where the deformation map $w_k(z,\bar{z})$ and Piola stress function $\chi_k(z,\bar{z})$ are given by

$$iw_k(z,\bar{z}) = \alpha_k\phi_k(z) + i\overline{\psi_k(z)} + \frac{\beta_k z}{\overline{\phi_k'(z)}},$$

$$\chi_k(z,\bar{z}) = 2i\mu_k\left[(\alpha_k-1)\phi_k(z) + i\overline{\psi_k(z)} + \frac{\beta_k z}{\overline{\phi_k'(z)}}\right], \ \text{for} \ k=1,2. \tag{1}$$

Equation (1) gives rise to the following Cartesian expressions for the stress and displacement fields

$$w_k(z,\bar{z}) - z = (u_1+iu_2)_k,$$

$$\chi_k(z,\bar{z})_{,1} = (\sigma_{22}-i\sigma_{12})_k, \ \ \chi_k(z,\bar{z})_{,2} = (-\sigma_{21}+i\sigma_{11})_k, \ \ k=1,2. \tag{2}$$

Assuming that the inclusion is imperfectly bonded to the matrix along the inclusion boundary curve ∂D_1 and utilizing the notation of [3], the imperfect interface conditions are given by

$$||\sigma_{rr}+i\sigma_{\theta r}|| = 0, \ \ \sigma_{rr} = m(\theta)\,||u_r||, \ \ ||u_\theta|| = 0, \ \ z\in\partial D_1, \tag{3}$$

where $m(\theta)$ and $n(\theta)$ are two non-negative imperfect interface parameters and $||.|| = (.)_2 - (.)_1$ is the quantitative jump across ∂D_1. By using the technique of analytic continuation on the continuity of tractions, continuity of tangential displacements, and radial stress-displacement condition the problem is reduced to the determination of a single analytic function $\phi_1(z)$ from an ordinary differential equation with variable coefficients which has the following general solution

$$(4)\qquad \phi_1(z) = (\frac{z}{R})^{2\Omega}\left[(\frac{z}{R})^s - \rho^*\right]^{\frac{\lambda\Omega\eta}{s}}\left[(\frac{z}{R})^s - \frac{1}{\rho^*}\right]^{\frac{-\lambda\Omega\eta}{s}}\int_{R\rho_1}^{z}(\frac{t}{R})^{-2\Omega+1}\left[(\frac{t}{R})^s - \rho^*\right]^{\frac{-\lambda\Omega\eta}{s}}\left[(\frac{t}{R})^s - \frac{1}{\rho^*}\right]^{\frac{\lambda\Omega\eta}{s}}\frac{P(t)}{\frac{t}{R}}dt,\ \ z \in \overline{D_1},$$

where Ω, η are material parameters and λ is a function of ρ^*, which characterizes the degree of interface imperfection along ∂D_1 and $P(t)$ contains the $s+1$ undetermined coefficients from the power series expansion of $\phi_1(z)$ which are given by the consistency condition of $\phi_1(z)$ at $z=0$ and certain other auxiliary conditions.

3. Results

Using (4) we may calculate the mean stress and contrast it to the homogeneous analog which yields the following

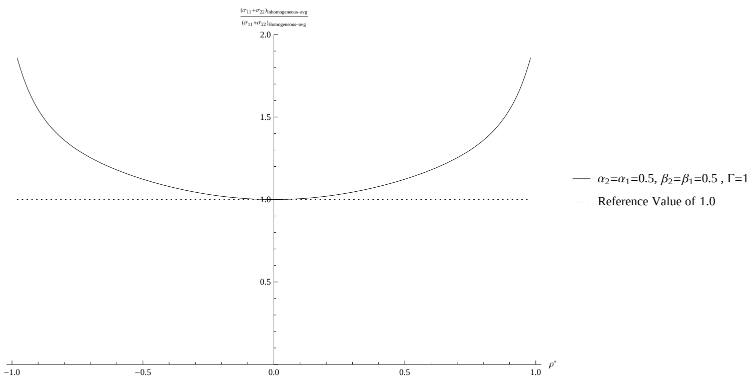

Figure 1. Ratio of inhomogeneous to homogeneous average mean stress on ∂D_1 for the remote loading $\sigma_{11}^{\infty} = 0, \sigma_{22}^{\infty} = 10^3, \sigma_{12}^{\infty} = 0$

Figure 2 clearly demonstrates that the inhomogeneous interface parameter ρ^* has a significant effect on the estimation of the average mean stress on the inclusion boundary and at its peak reaches an error of 80 percent.

4. References

[1] Fritz John. Plane Strain Problems for a Perfectly Elastic Material of Harmonic Type *. *Communications on Pure and Applied Mathematics*, XIII:239–296, 1960.

[2] C Q Ru. On complex-variable formulation for finite plane elastostatics of harmonic materials. *Acta Mechanica*, 234:219–234, 2002.

[3] L J Sudak, C Q Ru, P Schiavone, and A Mioduchowski. A Circular Inclusion with Inhomogeneously Imperfect Interface in Plane Elasticity. *Journal of Elasticity*, pages 19–41, 1999.