THERMAL STRESSES IN AN ELASTIC ISOTROPIC SPACE WITH AN ANTICRACK UNDER SYMMETRIC TEMPERATURE LOADS

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1. Introduction

The problem of thin rigid inclusions (anticracks) occurs frequently in various engineering applications [1-2]. The investigations of the strength of a solid weakened by these defects in thermal environments are based on the solutions of the corresponding thermoelastic problems. A method of solution to the antisymmetric problem for determining the steady-state thermal stresses in a space induced by an insulated rigid sheet-like inclusion of arbitrary shape under vertically uniform heat flow at infinity has been given in [3-4].

It is the purpose of this paper to present solutions to the symmetric case when the anticrack surfaces are exposed to the prescribed symmetric temperatures. It is stated that the problem can be considered as the counterpart of the corresponding mechanical problem [5]. The equations are solved by a general (thermal) potential method. A typical application to the case of a circular anticrack under uniform-temperature load (see Fig. 1) is presented. In this case a new complete solution expressed in elementary functions is obtained and analyzed from the point view of initiating fractures near the edge of the inclusion.

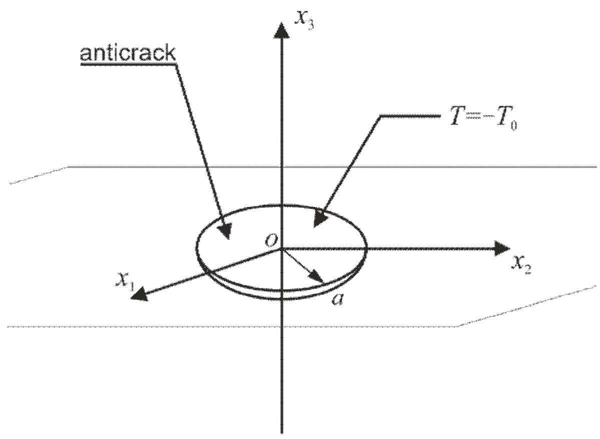

Fig. 1. A circular anticrack in an elastic space subjected to a constant temperature

2. Method and results

A traditional two-staged method of solution will be used. Using the symmetry conditions, first we need to solve a mixed boundary-value problem of heat conduction in a half space with the applied temperature over the anticrack surface. Secondly, we search for the solution to thermoelastic equations at the already known temperature field and with some mechanical anticrack boundary conditions. The governing 2D singular integral equations are derived for a planar anticrack of arbitrary shape S in terms of the shear stress discontinuities across inclusion $\sigma_{3\alpha}^{+} - \sigma_{3\alpha}^{-}$, $\alpha = 1,2$ as follows:

$$\begin{aligned}
&\frac{1}{2\pi\mu}\iint_S \left\{ \frac{\sigma_{31}^{+}(\xi_1,\xi_2)}{\sqrt{(x_1-\xi_1)^2+(x_2-\xi_2)^2}} \left[1-\kappa \frac{(x_2-\xi_2)^2}{(x_1-\xi_1)^2+(x_2-\xi_2)^2} \right] + \right.\\
&\left. +\kappa \frac{\sigma_{32}^{+}(\xi_1,\xi_2)(x_1-\xi_1)(x_2-\xi_2)}{\left(\sqrt{(x_1-\xi_1)^2+(x_2-\xi_2)^2}\right)^3} \right\} d\xi_1\, d\xi_2 = f_1^{\mathrm{THERM}}(x_1,x_2)-\varepsilon_1+\omega_3\, x_2 ,\\
(1)\quad & \qquad ,\\
&\frac{1}{2\pi\mu}\iint_S \left\{ \frac{\sigma_{32}^{+}(\xi_1,\xi_2)}{\sqrt{(x_1-\xi_1)^2+(x_2-\xi_2)^2}} \left[1-\kappa \frac{(x_1-\xi_1)^2}{(x_1-\xi_1)^2+(x_2-\xi_2)^2} \right] + \right.\\
&\left. +\kappa \frac{\sigma_{31}^{+}(\xi_1,\xi_2)(x_1-\xi_1)(x_2-\xi_2)}{\left(\sqrt{(x_1-\xi_1)^2+(x_2-\xi_2)^2}\right)^3} \right\} d\xi_1\, d\xi_2 = f_2^{\mathrm{THERM}}(x_1,x_2)-\varepsilon_2-\omega_3\, x_1 .
\end{aligned}$$

where $f_\alpha^{\mathrm{THERM}}(x_1,x_2)$, $\alpha=1,2$ are given from the solution of the thermal anticrack problem, and the constant κ is

$$\kappa=\frac{\lambda+\mu}{2(\lambda+2\mu)} \qquad (2)$$

with μ and λ being the Lamé moduli. Besides, $\varepsilon_\alpha\ (\alpha=1,2)$ and ω_3 stand for the corresponding displacements and the angle of rotation of the inclusion as a rigid whole that are determined from the equilibrium conditions.

As an example, a circular anticrack is analysed subjected to a uniform temperature $-T_0$. The analytical expressions for the relevant field quantities (e.g., the displacements, stresses, temperatures, heat fluxes in the inclusion plane) are given and discussed. This solution is also compared with that corresponding to a penny-shaped crack problem.

3. References

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